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THE MATHEMATICS TEACHER

Volume XLVI Number 6

October 1953

Modern Mathematics and the Gifted Student*

By MINA REES

Director, Mathematical Sciences Division, Office of Naval Research, Washington, D.C.

THIS PAPER is concerned with the world that confronts the gifted college student of mathematics upon graduation—a world in which employment opportunities are expanding, largely as a result of new emphases in certain fields of mathematical research since the end of World War II. In this article I will describe some of these fields of research, and I will try to indicate very briefly certain elementary aspects of the current activity in mathematics that might challenge the interest of gifted high-school boys and girls. Because I have been asked so frequently about opportunities for women, I shall point out certain mathematical careers in which women have been particularly successful.

It may be worth mentioning, in passing, the mathematics program of the Office of Naval Research with which I have been associated since its founding in 1946. This office has been closely identified with major recent developments in mathematical research. Its initial commitment to a strong program in mathematics reflected the effectiveness of the mathematicians who were engaged in scientific war work. The expansion of its mathematics program in scope and influence resulted from the excellence of the work and from its demonstrated importance to the Navy; the concomitant expansion of the number of professional openings in the world of business and industry for students with mathematical training reflects the fact that mathematics is constantly demonstrating on a broader front its substantial contributions in large segments of our economic life. It is the duty of teachers and research mathematicians and administrators to aid young people to discover the richness and the variety of mathematics and to seek careers that will provide intellectual as well as other rewards. I am concerned that some of the most exciting mathematical ideas of our times are not reflected in our educational scheme and that not enough is being done to enable our teachers to exploit these new fields for the benefit of their students. Much is made in the teaching of mathematics of the inspiration which may be derived from the history of mathematics and from the ideas of the Greeks and Egyptians. I would propose that there are many equally challenging ideas in the his-

* Based on a speech given at a meeting of the National Council of Teachers of Mathematics, held at Atlantic City, New Jersey, April 10, 1953.

More detailed information about opportunities for college graduates with mathematical training may be found in the pamphlet "Professional Opportunities in Mathematics" reprinted from the *American Mathematical Monthly*, LVIII, January, 1951. A revision of this pamphlet is being prepared.

EDITOR'S NOTE: Dr. Rees has assumed the duties of Dean of the Faculty, Hunter College, New York, as of September 1, 1953.

tory of our own times which may be reflected in our courses and in the extra-curricular intellectual contacts of our students. .

The postwar years have seen a flowering in America not only of work in pure mathematics—and America is now one of the world leaders in this field—but also of work in mathematical statistics, in electronic computers, in what I call classical applied mathematics (a study of continuum mechanics by analytical methods), in certain aspects of mathematical economics, and in operations research. Certain of these are widely represented in our university programs, some have only begun to percolate into the university curriculum, but few aspects of these subjects are in any substantial sense available to high-school students (except possibly through science fiction) so that even the most alert and interested boys and girls may not suspect what an exciting subject mathematics is. I shall indicate some features of the more applied fields that I think would challenge the interest of young people. These are some of the fields which hold the greatest promise for jobs after college.

First let me mention the role of mathematical statistics in industry. It has many facets. For example, there has been an increasing emphasis in industrial and government laboratories on the statistical design of experiments—and this is a field in which women have been particularly effective. Recently, for example, Miss Besse B. Day, a statistician attached to the U. S. Naval Engineering Experiment Station at Annapolis, described to the directors of all the laboratories operated by the Navy's Bureau of Ships the role which the design of experiments can play in maximizing the amount of information obtained from experimental work and in improving the quality of the information. As a result of this presentation Miss Day has been asked to consult with many of the laboratories of the Bureau of Ships and is assisting in improving experimental procedures in many cases. In another area widespread use of statistical methods and results has led to a constantly expanding effort in quality control and acceptance sampling. Quality control charts are regular tools in many progressive factories. It is noteworthy that the American Society for Quality Control, organized in 1946, has now about 6,000 members. By the setting of specifications the military establishment is at least partially responsible for the increasing emphasis on control of quality by statistical methods. A large-scale research effort to develop methods which will reduce the amount of inspection needed to insure high quality in the very extensive purchases made by the military departments was begun at the initiation of the Munitions Board a few years ago. Both in the control of quality at the factory and in the determination of acceptable material by the purchaser, students trained in statistics are particularly well equipped for supervisory work. In fact, the demand for well-trained statisticians appears to be a still expanding one. Ph.D.'s in statistics are in low supply, and even a modest amount of graduate work in statistics is extremely useful in getting a job in industry or government.

The second field that was mentioned above, the development of automatically sequenced electronic computers (the "giant brains" of a few years ago), has now reached the stage where there is an increasing opportunity for the employment of mathematicians in computing laboratories, and even on the sales staffs of major computing companies such as International Business Machines and Remington Rand. In April of this year IBM dedicated its new ultrafast electronic data processing machine. These great industrial companies have made very large financial commitments in the computer field, and this effort is accompanied by an expansion of a scientifically trained sales staff and of the staff needed to run large-scale computing establishments. The type of training needed by this latter kind of staff is extremely varied. There are the coders for

whom high-school training is probably sufficient, the programmers who need a good bachelor's degree in mathematics, and the analysts who require a very broad background in mathematics and related subjects. We should perhaps note the fact that the deliberate emphasis which was laid some years ago upon the expanding need for research and training in numerical analysis is now reflected in an increasing number of college courses in this general field. Several large computer laboratories are now going into operation. In almost every case I have been requested to help in recruiting a well-trained staff. This is surely a field in which employment opportunities will be on the increase during the next few years for persons with advanced training along relevant lines.

It would certainly seem worth while to introduce high-school students to some of the considerations which become critical when high-speed computers are used in mathematical analysis. For example, the introduction of binary arithmetic along with a discussion of the decimal system would give students familiarity with some of the basic notions useful in computer technology; the solution of simultaneous linear equations by iterative methods would introduce a point of view that is fundamental in many of the problems for which large-scale computers are used; and a discussion of the extent to which the handling of round-off errors can be decisive would warn students of some of the distinctive problems that arise in modern numerical analysis. What high-school boy or girl would not be challenged by the following problem involving round-off to three significant digits:

$$(.815\times .127)\times\frac{1}{.127}=.104\times\frac{1}{.127}=.819$$

$$\left(\frac{.815}{.127}\right)\times .127=6.42\times .127=.815$$

This illustrates a general principle proved in 1947 by Professor John von Neumann and Dr. H. H. Goldstine of the Institute for Advanced Study as a detail of their basic paper "Numerical Inverting of Matrices of High Order."[1] Clearly, great care must be observed in numerical work in the handling of seemingly trivial questions such as the order in which operations are performed, since even a slight loss of accuracy in a single operation may cause complete loss of significance in the final result if thousands or millions of operations are to be performed.

Another field which has taken on new vigor within the last few years involves certain aspects of mathematical economics. The new activity has arisen partly as a result of stimulation by the military establishment, but I anticipate that within a decade more activity will be reflected in industry. The problems have to do with the programming of production to meet consumer needs, and the effort is toward securing a more economical operation. The mathematics involved is for the present largely theory of games and combinatorial analysis, but studies are in progress to provide for the exploitation of electronic computers to improve supply operations and to maintain improved records of consumption and demand. Here it is fruitless to try to estimate the extent of job opportunities, but I consider that this is a field which may well be watched within the next few years.

The field of operations research has been developing since the war, and all the military departments, as well as the administrative structures of many forward-looking industrial organizations, have instituted rather extensive activities in this field. Operations research has been defined as the application of the techniques of the physical sciences to the study of the operations of war and peace. Clearly, if the methods of operations research are those of the physical sciences, they must involve quantitative analysis and they

[1] *Bulletin, American Mathematical Society,* LIII (November, 1947), 1021–1099.

must rely heavily on the tools of mathematics. In particular, probability theory is used extensively. A brief example of non-military operations research may help to clarify its nature.[2] Recently a study of hotel reservations was made which resulted in a set of rules that is now receiving a year's tryout by a hotel chain to see what bugs develop and what improvements result. The problem arises because at any given time a hotel has a definite number of guests in residence and has a definite number of reservations for a week from that day, for instance. How many more reservations should the booking clerk accept? If he turns away all reservations when he has his hotel full on paper, he will never have a full hotel, for about one out of five reservations never shows up. On the other hand, he cannot accept all reservations, for occasionally he will be called on to house more people than the hotel has room for. He must somehow balance the gain in having a larger number of rooms rented against the loss in good will of occasionally failing someone who has been given a reservation.

As soon as the hotel manager has decided how much such a failure costs the hotel, the other factors in the analysis are straightforward probability analysis. The computation of the mean stay of a guest and its relation to the check-out rate of guests already present, the curve for reservation cancellations, the dependence of fluctuations on time of week and so on—all these can be reduced to a fairly simple set of rules for the booking clerk to follow in making his day-by-day decisions.

Some of the newer branches of mathematics have even more direct connection with the problems of operations research than probability theory. Communications theory, as developed by Shannon and others, should become a very powerful tool for analyzing the flow of orders in an organization, for example; and the theory of games which provides for the specific inclusion of aspects of competition in the computation has been worked on intensively by some university and some military operations research groups. Thus, the training for operations research should include fairly sophisticated mathematics, but it should also have a broad base in the physical and social sciences. Most strikingly necessary for the success of a man operating in this field is the ability to work on a team in collaboration with others trained in different disciplines. There are some beginnings of plans for instituting Ph.D. training in operations research; already certain institutions are giving some courses in this general field.

I have not discussed what I have called classical applied mathematics because this is most appropriately thought of in connection with jobs in industrial or government laboratories. Increasingly, college students in mathematics are looking toward jobs in industry; and many more industries are looking toward the colleges to provide them with students trained in the sciences, and among them mathematics. The aircraft industry is absorbing many mathematically trained scientists, and I have already mentioned the computer field. In actuarial work and in operations research those who succeed are rather like engineers and lawyers who become executives. Their success is due less to the specific background of education out of which they come than it is to a combination of talents and a broad educational and business experience which they bring to their job. In actuarial work, a rather broad knowledge of economics and business practice is needed; in operations research posts, a fairly sophisticated training in the social as well as the physical sciences.

In industrial laboratories that employ professional mathematicians, there are essentially two grades of such employees: The research laboratories that employ persons with only an A.B. (or a B.S.) are

[2] Paraphrased from an article by Philip M. Morse, "Operations Research, What Is it?" *Journal of Applied Physics*, XXIII (February, 1952), 165–172.

usually looking for computers, and these are usually women. (In fact, in such laboratories there are few opportunities for women above the level of computers.) There are, however, new industries, like the electronic computer field, and certain other phases of the electronic industry, where an A.B. can find a job, and this may be true even if the one referred to is a woman. But here, as in most other laboratory work, the mathematician, to be worth his salt, should have some experience with and feeling for engineering practice (or at least some understanding of the outstanding facets of the physical sciences). In the second category of job in industry, a Ph.D. in mathematics is a *sine qua non*—but the requirement that the mathematician shall have some understanding of engineering problems still holds. The situation is roughly like this. The engineers who need mathematical help are themselves rather expert in the relatively well-known branches of mathematics like differential equations and probability theory. They get along very well without a mathematician until they hit a spot which is not well handled in the conventional way. Then they need a man who, understanding their problem, can find a mathematical model which may be quite unconventional but which will yield to possibly sophisticated, but usually relatively simple, mathematical treatment. It is seeing through the problem, and getting an adequate model that is usually the really tough part. And to handle this part successfully requires a mature and well-educated mathematician.

It is characteristic of the engineering approach that a real problem must be solved by whatever techniques and approximations and experiments may be necessary. The engineer does not have the luxury of the pure mathematician (and this luxury is equally characteristic of the theoretical physicist and other pure scientists) of defining his problem and setting up his boundary conditions so that the problem will yield a solution. Nor can he look in the back of the book for the right answer. The engineer must take problems which arise from an actual physical environment and somehow find a useful answer, whether the methods used are rigorous or not. And the mathematician who assists the engineer must have the necessary flexibility and imagination and technical command of his subject to provide a solution within this framework.

Much of what I have said concerning industrial laboratories is equally true for government laboratories. Again, persons broadly trained and with considerable experience in analysis are apt to be most valuable. However, it should be pointed out that the government environment is on the whole more favorable to women, since it is relatively harder to induce able people to work for the government than it is to induce them to work for industry. This is the environment in which women have the best chance.

I have said nothing of teaching, which is at once the most rewarding and probably the least glamorous of the activities which attract college graduates in mathematics. It is surely the field which should continue to receive the largest number of our enthusiastic and well-trained mathematicians. But it is also the field that is best known to most readers of THE MATHEMATICS TEACHER.

You will see that all this adds up to a recognition that college graduates with mathematical training, and particularly Ph.D.'s with mathematical training, have important roles to play in industry and in government as well as in the universities. It is critical throughout our considerations, however, to recognize that our real contributions as teachers lie in the direction of providing real mathematics and not a watered-down version to our students, and in seeing to it that they acquire as broad a base in the sciences and in the humanities as is feasible. At the secondary school level the obligation of the mathematics teacher is particularly acute. I was sitting next to a distinguished psychologist

at a luncheon the other day, and I was heartened to hear him remark that the welfare of all the sciences depends critically on an improvement in the mathematics curriculum of our secondary schools. Here is an area where the college teachers and the high-school teachers have a great common undertaking, which it is good to know that the university research workers in mathematics are now keenly appreciating. Though we have by no means solved the problem of producing better mathematics textbooks at the college level, urgent and conscientious work is now going on in this area. I know that your organization is also heavily committed to working for the improvement of the mathematics curriculum so that high-school and college students can learn not only routines but also something of the quality of modern mathematics.

We need books written in a way that will appeal to young people, that tell of some of the exciting successes, and that particularly explain some of the unsolved problems. Our students need contact with research people so that they will know that mathematics is alive. I consider it a dreadful failure of our educational process that so many students graduate from college utterly unaware that new and vital results in mathematics are being found daily. Our students need to appreciate the great scope and depth and variety of mathematics, and they need to know that the most effective mathematician is a cultivated person with a broad education outside mathematics. The future is, I am sure, rich in opportunity for well-trained mathematicians; not only for those who carry the torch of devoted and imaginative teaching and for those who experience the rewards of mathematical research, but rich in opportunity also for those who accept the challenge of modern developments in the many phases of applied mathematics and who seek careers in government and industry.

HAVE YOU READ?

Carnahan, Walter H., "More Math for More Students," *N.E.A. Journal*, April 1953, p. 219.

How well are we as a nation meeting the individual and social demands for mathematical training? "We aren't," says Mr. Carnahan. To be a vegetable you need no mathematics, but as a competent, participating citizen in the world of today your mathematical needs are increasing. These needs are considered in many aspects.

PHILLIP PEAK

Meade, Kenneth, "The Type of College Graduate Desired by a Manufacturing Industry," *School Science and Mathematics*, June 1953, p. 483.

This presents an answer to the question, "What constitutes the components necessary for a successful career in General Motors?" Since less than half of all college graduates employed by this company are engineers, the students in the Liberal Arts and other schools should have the opportunity for an educational program which develops these necessary components. Suggestions for such a program are presented in this article.

Reid, Constance, "Perfect Numbers," *Scientific American*, March 1953, p. 84.

The age-old question of "How many perfect numbers are there?" has not been answered by the National Bureau of Standards' Western Automatic Computor but it has opened the way to finding more of them. To test Euclid's expression $2^{n-1}(2^n-1)$ which is a perfect number when 2^n-1 is prime, SWAC took 48 seconds to show $2^{257}-1$ was not prime, while Lehmer spent 700 hours a number of years ago to make the same check. SWAC has now found five perfect numbers to add to the 12 previously known. The astronomical size of these numbers is appalling.

——— "Wall Street," *Coronet*, June 1953, p. 47.

Anyone who needs material to make the study of stocks and bonds more realistic in the eyes of the student, will find this word and picture story of Wall Street a valuable aid. A sequence of pictures and paragraphs shows the development of Wall Street from the lowly cowpath to the great market place of finance. The import and purpose of such a colossal auction place is well portrayed.

What Is Wrong with School Arithmetic?

By Gladys Risden
Vermilion, Ohio

Arithmetic is the science of putting quantities together and taking them apart. Thousands of the children in our schools have never had a chance to find out that school arithmetic has anything to do with quantities. They are taught to juggle figures.

Lee looked at 3 red toy tractors and 4 blue ones and told me there were 12 tractors.

"Count them," I said.

"Oh," he answered, "I count 7, but there are 12. Three times 4 are 12. I learned that in school today. I know my multiplication good."

Kate said, "We learned a new kind of problems today—2 and 3 and 2 are 25."

I laid down 2 spoons and 3 spoons and 2 spoons. "How many spoons are there?"

Kate counted, "1, 2, 3, 4, 5, 6, 7. But that's the kind of arithmetic we had last year in the first grade. This year in the second we have 2 and 3 and 2 are 25." She showed me on her paper, marked A:

$$\begin{array}{r} 23 \\ \underline{2} \\ 25 \end{array}$$

I looked at Ned's arithmetic test paper, marked A: "$\frac{1}{4}$ of 8 are 2, etc."

I gave him 8 cookies. "You may keep a fourth of those and give a fourth of them to each of the other boys. But first show me how many in your fourth."

He held up 4. I shook my head. He held up 1. I shook my head. Then he just stood perplexed. There were two figures—1 and 4—and he had done something with each. He had no further ideas on the subject. Yet he had written "$\frac{1}{4}$ of 8 are 2."

Illustrations such as these fill my files. Children—average and superior I.Q. children—go to school and learn tricks with figures but they never find out "how many."

Lacking awareness of "how many," these children never discover that one fact bears relation to another. Children who don't miss the combinations "6 and 6" and "7 and 7," or "3 and 3" and "4 and 4" regularly miss "6 and 7" and "3 and 4."

Ted, who has been fortunate in having a different kind of school arithmetic, said to his cousin, Jack, "Why, can't you see 6 and 7 would be 1 more than 6 and 6, 13. Or you could think 1 less than 7 and 7, 13. Or you could think 10 and 3. See? Just think 3 away from the 6 to make the 7, 10. That would change it to 10 and 3. Or you could think 5 and 5 and 2 and 1, 13." Ted rattled these off as fast as he could talk.

Ted's eight-year-old brother decided to find out the sixes by "his own self" the other day: "*4 sixes,* 2 sixes and 2 more sixes, 12 and 12, 24; *3 sixes,* 12 and 6, 18; *5 sixes,* 3 sixes and 2 sixes, 18 and 12, 10 and 10 and 2 and 8, 30; *6 sixes,* 30 and 6, 36; or 3 sixes and 3 sixes, 18 and 18, 4 less than 2 twenties, 36; or 4 sixes and 2 sixes, 24 and 12, 36."

Ted's eight-year-old brother didn't need objects to work out the sixes. But when he worked out the threes he did. His teacher helped him to arrange the groups of groups:

*** *** *** ***	*4 threes;* same as 2 threes and 2 threes.
*** *** *** *** *** *** ***	*7 threes;* 4 threes and 3 threes or 3 less than 4 threes and 4 threes,
or	
*** *** ***	5 threes and 2 threes, or
*** *** *** ***	3 threes less than 10 threes.
	etc.

Ted and his brother attend a school that starts them with putting quantities to-

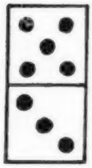
gether and taking them apart; it guides their growth in thinking them *together* and *apart.*

"But what is so different about this?" you say. "Haven't schools been using objects from times immemorial?"

Using objects, yes, but using them to illustrate a dehydrated fact, not to learn to think quantities *together* and *apart.*

To illustrate the difference:

Andy is going on seven. He has been "learning" (?) the easy addition combinations. A domino card was shown the class.



"Five and 3 are 8," said John, who was waving his hand. "Five and 3 are 8," the others chanted. The teacher placed the card up on the blackboard ledge with the others there and said, "Now I want you to write the answers to these combinations on this paper. If you don't know them, look up here, find the card, and count."

Andy got F's for six weeks; then he began getting A's. Mom was delighted.

"Don't get too high, Mom," said young Andy reluctantly. "Linda Lou has been telling me the answers." Mom brought young Andy over to me at that point.

I showed Andy 3 buckeyes. "How many?" I asked.

"One, 2, 3," said Andy.

M-m-m. So Andy didn't really *know* 3. *Knowing* it, he would have seen 3 at a single glance. I arranged several groups: 1 *three,* 1 *two,* 1 *four,* 1 *five.*

"Show me the pile that has 3 in it," I said.

Andy started with the pile nearest him and counted each by ones until he found one pile that stopped at 3.

I dropped a dozen buckeyes on the table with apparent carelessness but actually leaving two groups of 3 each standing apart so as to be easily recognizable.

"Give me 3 buckeyes."

He started with the one nearest him, counting 1, 2, 3, and gave me 3. He couldn't see 3 even when it was standing apart begging to be seen.

If the handicap of thus seeing 3 as 3 *ones* instead of as 3 isn't apparent to you, try this.

Look at Figure 1 and tell how many zeroes as quickly as possible:

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

FIG. 1

Now do the same with each of the figures below:

```
00 000     00 000       00 00
00 000     00 000       00 00
00 000     00 000       00 00
00 000     00 000       00 00
00 000     00 000       00 00

00 000     00 000       00 00
00 000     00 000       00 00
```

FIG. 2

I tried the above on ten-year-old Ruth (I.Q., average). Her answer came in three seconds by the stop watch, "98."

"Tell me how you thought it."

"I could see 7 tens and 7 fours, 70 and 28. And I can see 5 fourteens and 2 fourteens, 50 and 20—that's 70—and 20 and 8—that's 98. You could see 4 fourteens and 3 fourteens, too, like this—" and she rearranged the zeroes as shown in figure 3.

```
000 0000     0000 000
000 0000     0000 000
000 0000     0000 000
000 0000     0000 000

000 0000     0000 000
000 0000     0000 000
000 0000     0000 000
```

FIG. 3

"I can see 2 forty-nines, 2 less than 100, 2 less than 2 fifties," she explained. "And take one off each of these fours and we'd have 7 sixes and 7 sixes, 84, and put the ones back on, 14 more, 98."

I wanted to see whether Ruth was as yet independent of objects for seeing. "How much are 8 seventeens?" I asked.

"136," she answered without more than two seconds' thought. "I know because 8 seventeens are 8 tens and 8 sevens, 80 and 56. And another way—2 more seventeens would be 10 seventeens, 170; 34 from 170, 30 from 170 would be 140, and 4 more, 136. And another way—" But we haven't space to continue.

Ruth is learning to take groups apart and put them together. She began by using objects but she has reached independence. Does Ruth have trouble with "story problems?" The answer is *no*. Does she make those little errors such as calling 8 sevens 57? The answer is *no*.

What ten-year-old Ruth can do today we want eight-year-old Andy to do two years from now. So we begin by giving experiences with twos and threes until he can recognize "twoness" and "threeness" at sight. Then we begin use of these in helping identify larger quantities. We toss out 4 buckeyes. We separate into 3 and 1. "Three and another 1, 4," we say. "Now you take 4 apart another way. Can you break it up into twos?" We have him select fours, not by counting ones (we want to break up that firmly rooted habit of seeing ones) but by taking 2 and another 2 (recognizing at sight, not counting) or 3 and 1 more. We toss out groups for him to recognize instantly.

"I don't have to count. I just see them," he says proudly on the third day.

So we are ready for the next step up the climb to the child's own generalization; 2 and 3 are 5, etc. We "say with figures." Tossing down 5 buckeyes so they fall as 3 and 2, we say, "What do you see?" "Five. Three and 2 are 5." Saying it with figures, Andy writes

$$\begin{array}{c} 3 \\ \underline{2} \\ 5 \end{array}$$

Three more days and he can "say with figures" any arrangement of 4, 5 or 6 units that he sees. No drill yet. The time has not come for that.

Now is the time to begin thinking with images.

"Three kids are on the school bus and 2 more get on," we say. "How many are on the school bus now?"

No response. We took two steps instead of one. Back up a step.

"Three Gova kids (Andy and his brother and sister) get on the school bus. Now 2 Smiths get on the school bus. How many on the school bus?"

"Five." Answer comes quick as a toad can flick its tongue.

"One Peters kid gets on the bus. Now how many?"

"Six."

"Now we come home. Six kids on the bus when they turn down this road; 1 gets off at Peters."

"Now there are 5."

"Two get off at Smiths."

"Now there are 3."

"Three get off at Govas."

"Now the bus is empty."

Drill? Not yet. But perhaps he can think about kids on the school bus who aren't the kids on his road. So we have any school bus and any kids.

"Four kids are on the school bus and 2 more get on."

"Six."

Next step.

We gather eggs and spend pennies and eat apples. On the seventh day Andy no long had to visualize personal experience. He had gone far on the road of abstraction to generalization. Next step.

$$\begin{array}{cccccc} 3 & 4 & 3 & 2 & 4 & 4 \\ \underline{2} & \underline{2} & \underline{3} & \underline{2} & \underline{+2} & \underline{-2} \end{array}$$

"Now you think about finding Easter eggs."

"I found 3 eggs in the barn and 2 under the porch. I found 5 eggs." He writes 5. "This says I found 4 eggs and ate 2. I have 2 left." He writes 2 under 4 minus 2.

Drill? Not yet. This is a big difficult step. Not until a week later is he able to look at $\begin{array}{c}2\\ \underline{3}\end{array}$ and say 5. Now he is ready for drill. But he doesn't need drill. He knows them all.

He jumps off the school bus, proudly waving his paper. "Every one right, Mom. And Linda Lou didn't have to tell me the answers, Mom. I knew the answers my own self."

Twenty years ago I read in THE MATHEMATICS TEACHER a statement in which a mathematical wizard attributed his "genius" to childhood play. He would sit by the hour, he said, with a pile of pebbles, putting them together and taking them apart, breaking them down into groups he could think with ease.

Miss Diane Bellman, an exchange teacher from England, teaching third grade in Elyria, Ohio, recently, says, "These children see numbers just as ones. They can't think with ones. And so they make many, many little mistakes, such as calling 6 and 7, 14—and calling 7 and 7, 14, in the next problem. In England we teach children to see groups. They understand. American children remember—when they don't forget."

A teacher, superior in intelligence, said to me, "I hated arithmetic in school. I got by somehow by sheer memory and a lot of guessing when memory didn't serve. But I never *understood*. I learned to be afraid of arithmetic because I was afraid I would forget and if I forgot there was *nothing* I could do about it. I was so afraid I chose kindergarten teaching so I wouldn't have to teach arithmetic. I was afraid until I took this course. Now I understand, I can think. I don't have to *remember*, I *know*." The course to which she was referring was one in which grouping was used for thinking quantities together and apart.

An editor writes, "I have always hated arithmetic and remember that once when I was a child I ran home from school and hid under my mother's bed because I hadn't prepared my arithmetic lesson. I couldn't prepare it because I didn't understand what it was all about. It was not, in fact, until later years that I came myself to have a working knowledge of arithmetical computations and how to make them work for me instead of the other way around."

A successful business man said, "All the arithmetic I know today I learned after I left school. In school I just memorized a lot of words that meant nothing to me. I forgot them all before I was through high school. Then I began from scratch and learned by simply thinking 'how many.' "

What is wrong with school arithmetic?

The mathematicians say, "Not enough drill." Strange, isn't it? The mathematicians value mathematics as a thinking discipline, yet they advocate learning as the parrot learns—by rote.

The progressives say, "Not enough experience." And their children count 1, 2, 3, 4 to find this and that. And they write down their figures and add with no more thought to the difference in value between a 2 in ten's place and a 2 in one's place than does the child in the traditional school have.

No, they are both wrong. What is wrong with school arithmetic is too little understanding, too little use of the higher mental processes, too much finding answers, not enough finding "how much" and "how many," too much repetition of someone else's generalization (meaningless to the one who has had no opportunity to make it for himself) and not enough opportunity to experience, to abstract and to generalize for oneself.

The only generalizations that work for a person are the ones he makes for himself.

Let's give our young ones a chance to *put quantities together* and *take them apart*. Let's talk with them about what they are doing, guiding them in "saying" what they find out. Let's make school arithmetic a science of putting numbers (no, not numbers, quantities) together and taking them apart.

MINUTES OF ANNUAL BUSINESS MEETING

National Council of Teachers of Mathematics
Ambassador Hotel, Atlantic City, New Jersey
Saturday, April 11, 1953

The meeting was called to order by the President, John Mayor, at 8:06 A.M.

Mr. Mayor introduced the newly elected officers, who are as follows:

Glenn Ayre, Vice President for College Mathematics
Mary C. Rogers, Vice President for Junior High School Mathematics
Howard Fehr, Member of Board of Directors
Phillip Jones, Member of Board of Directors
Elizabeth Roudebush, Member of Board of Directors

Mr. Mayor introduced the following appointments by the Board of Directors:

Henry Van Engen, Editor of THE MATHEMATICS TEACHER, to assume office June 15, 1953
Houston Karnes, member of the Board of Directors, to succeed Henry Van Engen, who will resign June 15, 1953
Agnes Herbert, Recording Secretary of the Board of Directors

M. H. Ahrendt, Executive Secretary, reported that progress had been made toward the membership goal of 10,000, the number of paid memberships on April 1 being 9,453. He stated that sales of yearbooks and other publications of the Council had increased. He invited members of the Council to visit the office of the Executive Secretary in Washington, D.C.

E. H. C. Hildebrandt expressed thanks to the associate and department editors of THE MATHEMATICS TEACHER for their cooperation during his term as editor. He stated that reader polls which he had conducted showed all departments were meeting needs of Council members, but that it was essential that services to elementary teachers be increased, and that it would be desirable to provide material for the use of secondary-school pupils.

George Hawkins introduced the following resolution and moved its adoption:

THAT members of the National Council of Teachers of Mathematics assembled in this annual business meeting express our appreciation to our retiring editor of THE MATHEMATICS TEACHER, E. H. C. Hildebrandt, for his inspirational leadership and his untiring effort which have produced under his editorship our excellent journal, and that we now look forward to his more leisurely participation and help in all the activities of the Council.

The motion was carried unanimously.

Henry Syer, Chairman of the Committee on Publications of Current Interest, reported that the Committee was planning four types of publication to be sold for prices ranging from 10 to 25 cents: (1) materials to provide background in mathematics for teachers; (2) materials for students for enrichment purposes; (3) a "how to" series for teachers; (4) miscellaneous service publications.

Miss Agnes Herbert, Chairman of the Committee on Place of Meetings, announced the following schedule of meetings:

August 24–26, 1953, Kalamazoo, Michigan
December, 1953, Los Angeles, California
Spring, 1954, Cincinnati, Ohio
Summer, 1954, Seattle, Washington
Spring, 1955, Boston, Massachusetts

W. A. Gager announced that the Council will hold a meeting on June 29, 1953, in Miami Beach, Florida, in connection with the summer meeting of the National Education Association.

Mr. Mayor urged the necessity for geographic distribution of meeting places and pointed out the need for invitations for future meetings. Mr. Syer urged the consideration of college campuses as meeting places, in the interest of economy.

Mr. Van Engen, Chairman of the Committee on a Proposed Publication for Elementary Teachers, reported that on April 10, 1953, the Board of Directors had voted to establish a publication for elementary teachers, to be devoted mainly to arithmetic in grades 1–6, and had appointed a committee to propose names for the editorship in order to begin publication as soon as possible. W. D. Reeve commented on the desirability of obtaining financial subsidies in order to enlarge the publishing activities of the Council. F. L. Wren asked what the relation would be between subscriptions to the new journal and membership in the Council. Mr. Van Engen replied that it was the consensus of the Board that it would be desirable to have this relationship the same as in the case of THE MATHEMATICS TEACHER, and the Board was studying the question to determine what changes in the by-laws would be necessary.

Mr. Van Engen, Chairman of the Research Committee, reported that, to stimulate research in the field of mathematics education, the Board of Directors had voted to offer an award of $1,000 for the best prospectus for a study of a learning problem in mathematics. The announcement of the award will appear shortly in THE MATHEMATICS TEACHER and other journals and will be sent to colleges and universities.

Miss Agnes Herbert, Chairman of the Committee on Relations with the National Education Association, offered the following resolutions and moved their adoption:

1. WHEREAS, In anticipation of the one-hundredth anniversary of the National

Education Association a Centennial Action Program has been planned, and

WHEREAS, The objectives of the program are desirable for the advancing and uniting of the teaching profession,

Be it resolved: That we approve and commend the principles of the Centennial Action Program and that the National Council of Teachers of Mathematics offer to cooperate in all appropriate ways to bring about its fulfillment.

2. WHEREAS, The offices of the National Education Association are housed in quarters never intended for office use and which are adequate for a staff less than half the size of the present staff, and

   WHEREAS, A five-million-dollar building and remodeling fund is being raised to provide a building worthy of housing the home of the headquarters of the teachers of the United States,

   Be it resolved: That we approve the purpose of the building fund and invite the members of the National Council of Teachers of Mathematics to participate in the ways outlined for promoting the program.

3. WHEREAS, The school children of war-devastated Korea are being taught in tents, barracks, warehouses, and unheated school buildings, and

   WHEREAS, The Korean teachers, though poorly fed, inadequately clothed, and severely underpaid, are carrying on courageously,

   Be it resolved: That we commend the reactivation of the National Education Association Overseas Teachers Relief Fund and urge the members of the National Council of Teachers of Mathematics to support this fund in order that the schools may be kept open and effective in Korea.

The motion to adopt these resolutions was carried.

Miss Herbert also proposed and moved the adoption of the following resolution:

WHEREAS, The host organization, the Association of Mathematics Teachers of New Jersey, and the teachers of Atlantic City have shown unusual courtesy and consideration to those persons attending this convention,

Be it resolved: That we express our appreciation and gratitude for all the courtesies shown and for so ably completing the unusual and necessary arrangements for the success of this meeting.

The motion to adopt this resolution was carried.

Harry Charlesworth proposed the following resolution and moved its adoption:

WHEREAS, The United States Office of Education has rendered distinguished and invaluable services to education, and

WHEREAS, The creation of the position of Specialist for Mathematics in Secondary Schools has been of great value to the cause of mathematics education in this country, and

WHEREAS, The educational activities of the federal government have been delegated to a multiplicity of offices, departments, and bureaus throughout the government structure, and

WHEREAS, The United States Office of Education has been severely handicapped in its activities by inadequate funds,

Be it resolved: That the National Council of Teachers of Mathematics express its appreciation for the service rendered by the United States Office of Education and respectfully urge (1) that this office be strengthened and supported by Congress in order that the services of this office may grow; and (2) that the educational activities of the federal government be coordinated and channeled through the United States Office of Education.

Mr. Ransom commented that the second clause of the resolution seemed to urge continuance of an office which may soon be replaced by some other agency, and moved to amend the resolution by striking out Clause 2. After discussion Mr. Ransom withdrew his motion to amend. Henry Swain then moved to amend the resolution by striking out Clause 2 and replacing it as follows:

(2) that the educational activities of the federal government be channeled through and coordinated by one agency.

This motion was carried and the resolution was adopted as amended.

Mr. Charlesworth moved that an appropriate letter conveying the sense of this resolution be sent by President Mayor to members of the Senate Appropriations Committee and the similar Committee of the House of Representatives, to the Commissioner of Education, and other interested persons in the Office of Education.

The motion was carried.

Miss Madeline Messner moved that on behalf of the National Council the Executive Committee express to the family of the late Professor Schlauch their appreciation of his many services to the National Council and the causes which it represents. The motion was carried.

The meeting adjourned at 9:19 A.M.

Respectfully submitted,
LENORE JOHN, *Secretary to the Board of Directors*

Membership Report

As all active members of the Council know, we have been working toward a membership goal of 10,000. We had hoped to reach this goal by the end of the 1952–53 school year. We did not quite reach it, but we did not miss it far. The following figures tell a thrilling story of the growth of the National Council of Teachers of Mathematics.

May 1, 1952	8733
February 2, 1953	9179
April 1, 1953	9453
May 1, 1953	9661
June 10, 1953	9800

It will require a determined struggle for us to hold these fine gains, however, during the fall months. About 80 per cent of our memberships expire during the summer and fall months. When the renewals come in late or do not arrive at all, the number of active plates in the membership files takes a sudden drop. Thus during the months of October, November, and December we "have to run fast just to stay where we are."

You can do two things to insure the continued growth of the Council: (1) When your own expiration notice reaches you, return it promptly with your remittance. (2) Try to sell as many of your "unenlightened" colleagues as you can on the benefits of joining the Council and reading The Mathematics Teacher. *Can we reach the goal of 10,000 during 1953?*

Since many membership counts have been made on May 1, we have given below, for comparison with the record of previous years, an accurate report of membership by states for May 1, 1953.

M. H. Ahrendt, *Executive Secretary*

MEMBERSHIP IN THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

May 1, 1953

State	*Individual*	*Institutional*	*Total*
Alabama	100	32	132
Arizona	27	13	40
Arkansas	94	15	109
California	265	179	444
Colorado	103	21	124
Connecticut	117	37	154
Delaware	35	5	40
District of Columbia	131	13	144
Florida	164	45	209
Georgia	88	34	122
Idaho	11	6	17
Illinois	685	109	794
Indiana	269	53	322
Iowa	163	46	209
Kansas	188	26	214
Kentucky	154	25	179
Louisiana	152	35	187
Maine	31	9	40
Maryland	157	29	186
Massachusetts	238	52	290
Michigan	256	88	344
Minnesota	182	53	235
Mississippi	62	22	84
Missouri	132	50	182
Montana	28	8	36
Nebraska	129	19	148
Nevada	2	4	6
New Hampshire	29	10	39
New Jersey	314	73	387
New Mexico	36	19	55
New York	571	176	747
North Carolina	125	42	167
North Dakota	24	11	35
Ohio	395	75	470
Oklahoma	101	45	146
Oregon	51	17	68
Pennsylvania	436	160	596
Rhode Island	30	8	38
South Carolina	55	40	95
South Dakota	22	8	30
Tennessee	103	44	147
Texas	333	124	457
Utah	26	10	36
Vermont	16	7	23
Virginia	199	50	249
Washington	57	37	94
West Virginia	84	13	97
Wisconsin	240	53	293
Wyoming	26	6	32
Totals	7236	2056	9292
U. S. Possessions	22	13	35
Canada	84	49	133
Foreign	97	104	201
Grand Totals	7439	2222	9661

MINUTES OF FOURTH DELEGATE ASSEMBLY

National Council of Teachers of Mathematics
Ambassador Hotel, Atlantic City, New Jersey
April 9 and 10, 1953

The Fourth Delegate Assembly of the National Council of Teachers of Mathematics was called to order by the Chairman, Mary C. Rogers. A breakfast preceded the meeting, at which time, Dr. John R. Mayor, President of the Council, welcomed the delegates to the convention. The four regional representatives were introduced, and they in turn introduced delegates from their respective areas.

Fifty-two delegates, nine alternates, and twelve special guests attended the Assembly sessions.

First Session of the Assembly

Certificates of affiliation were given to the following groups affiliated since April 18, 1952:

Delaware Council of Mathematics Teachers
Gary Council of Teachers of Mathematics
Texas Council of Teachers of Mathematics
Wyoming Council of Mathematics Teachers

Groups anticipating completion of affiliation in the Spring or early Autumn of 1953 were announced:

Michigan Council of Teachers of Mathematics
Missouri Affiliated Group of The National Council of Teachers of Mathematics
Association of Mathematics Teachers of New York State

Miss Rogers congratulated the Affiliated Groups organization for a most successful year of 1952–53. She attributed this success to:

1. The outstanding interest shown by the leaders of the Affiliated Groups, and their enthusiastic cooperation in all activities and services of the Affiliated Groups program.
2. The excellent leadership of the Regional Representatives.
   a. Their services to the groups in their areas.
   b. The Newsletters prepared with the assistance of their local groups.
   c. The helpful articles on Affiliation activities in The Mathematics Teacher.

Miss Rogers called attention to additional services given National Council by the Affiliated Groups organization:

1. Special programs for Affiliated Groups at all National Council meetings, prepared and directed by the Committee on Affiliated Groups.
2. Sectional meetings at these conventions, sponsored by local groups.

She supplemented this information with data from the 1952–53 Affiliation Renewal Reports.

1. The Affiliated Groups organization is now comprised of 32 State, 5 Regional, 6 County and 18 City groups.
2. Of the groups reporting local membership, 33 groups have 100 members or less; 12 groups have 101–200 members; 8 groups have 201–400 members; 3 groups have more than 400 members.

A report from the Washington Office was given by M. H. Ahrendt, Executive Secretary. He pointed out:

1. The Affiliated Groups can be of great assistance to the State Representatives in publicizing the programs and services of the National Council.
2. Membership in National Council on April 1, 1953, totaled 9,453—an increase of 604 over last year. The membership goal is 10,000.
3. Publication sales have increased since the establishment of the Washington Office.

Dr. Kenneth E. Brown, Specialist for Mathematics, U. S. Office of Education, spoke of the need for education in the armed services. Dr. Brown said the government had anticipated 3,000,000 men in service at one time. It is now thought the number will be cut to 1,760,000. The draft age will be 18½–19 years. Many of these service men have high-school diplomas, but cannot read or write well. On the other hand, some are sufficiently proficient in educational achievement to merit exemption for special scientific and technological training in connection with the Defense Program.

Dr. Brown requested suggestions from the delegates which might help answer these questions:

1. Why the inability to read on the part of so many?
2. What standards of achievement are reasonable at the various grade levels?
3. What can we do to help both the rapid and slow learners?

Dr. William A. Gager spoke on the purposes of the Delegate Assembly. The National Council can work most effectively through the Affiliated Groups. The Delegate Assembly is a preliminary Board meeting. It serves to bring the state and local organizations into sharp focus. The National Council tries to encourage cooperation and cooperative planning between groups through the exchange of ideas by the delegates. It is the responsibility of each delegate to report back to the local organization as soon as possible all business transacted and recommendations made.

Mr. Henry Swain suggested that there be more publicity concerning the aims and objectives of the Delegate Assembly and its services to the local groups.

Miss Ruth Lee Green asked for guidance on group organization—suggestions for the publication of local Newsletters, advice concerning

special research or group studies, aid in setting up more effective local programs.

Mr. Jackson Adkins asked whether certain decisions now made by the Board could not be made by the Delegate Assembly. Dr. Mayor replied that The National Council of Teachers of Mathematics Constitution indicates that the Delegate Assembly serves in an advisory capacity to the Board. Many recommendations made by the Assembly at earlier sessions have been accepted and activated by the Board. The Delegate Assembly is a liaison between the Council and the local groups. It is the responsibility of the Assembly to examine plans for the future and to make recommendations; to report activities and to make suggestions.

Dr. F. L. Wren suggested that a committee be appointed to prepare a Handbook for Affiliated Groups which would answer the questions raised by the Assembly. Dr. Carl Shuster moved the appointment of such a committee; Mr. Henry Swain seconded the motion; the motion was carried.

Dr. E. H. C. Hildebrandt spoke as editor of THE MATHEMATICS TEACHER. Some of the points he stressed were:

1. There is a need for closer cooperation among the various publication staffs and committees of the National Council.
2. The Affiliated Groups have received generous publicity in THE MATHEMATICS TEACHER.
3. Special attention has been given to the elementary teacher. There is a strong possibility of a special publication for elementary teachers in the very near future.
4. High-school students are reading our publication. A committee is currently studying the feasibility of the National Council's sponsoring a special publication for these students.
5. The Affiliated Groups can be of service to the National Council in setting up THE MATHEMATICS TEACHER Loan Libraries —to include past issues of THE MATHEMATICS TEACHER, out-of-print Yearbooks and similar publications.

Dr. Henry W. Syer, chairman of the Committee on Publications of Current Interest, urged more active and creative support of the work of his committee. Suggestions for articles come in "by the bushel basket," but *articles* are very much needed by the committee. To date, twelve manuscripts of the following types have been submitted:

1. Higher mathematics for teachers.
2. Enrichment materials for students.
3. "How to" suggestions and discussions on "how to do," "how to make," "how to use."
4. Service publications—surveys, tests, books, etc.

Dr. F. L. Wren, chairman of the Yearbook Planning Committee, explained the organization and rotation of membership on his committee. This committee has planned and been instrumental in the publication of the Twenty-first Yearbook. The Twenty-second Yearbook is on the way. The Twenty-first Yearbook, entitled The Learning of Mathematics—Its Theory and Practice, was edited by Dr. Howard F. Fehr and is now available to members of The National Council of Teachers of Mathematics at $3.00 per copy.

The Twenty-second Yearbook—entitled Emerging Practices in Mathematics Education —is being edited by Dr. John R. Clark. Dr. Clark reported to the delegates that the new book will include much helpful information concerning emerging practices in mathematics education; recommendations concerning curriculum adjustments; new practices in evaluation; in-service training of teachers—institutes, workshops, etc.; and the new emphasis on content and new subject matter, such as relativity and approximate computation.

Dr. Wren suggested that this yearbook would be helpful to new teachers, emergency teachers, and teachers not well informed. He further reported that the committee was recommending a series of five additional yearbooks:

1. Interpretations of Modern Mathematics for the High School Teacher.
2. Rethinking the Elementary Mathematics Curriculum, grades 1–6.
3. Rethinking the Mathematics Curriculum, grades 7–12.
4. Rethinking the Mathematics Curriculum in Teacher Training.
5. Evaluation of the Curriculum.

Dr. Francis Lankford will be the new chairman of the committee.

SECOND SESSION OF THE DELEGATE ASSEMBLY

The Friday morning session was opened by Miss Mary Rogers. Miss Rogers announced that the Committee on Affiliated Groups was studying the request for a Handbook Committee. This committee will be organized at the earliest possible time and notifications sent out by mail.

Miss Tremper of Michigan issued an invitation to attend the Kalamazoo Summer meeting, August 23–26, 1953, on the campus of Western Michigan College of Education. She presented the tentative program.

Dr. Phillip S. Jones, Chairman of the Committee on Cooperation of Mathematics with Industry, brought a report to the delegates from his committee. He paid tribute to Professor W. W. Rankin who instigated the work of this group and was for several years its chairman.

The Committee has planned five meetings for this year; it has a list of fifteen possible projects in mind. The plan of action will be decided by the committee and information disseminated to interested people as the work progresses.

Dr. Jones gave the five following examples of planned activity and invited the cooperation of the Affiliated Groups in developing and publicizing these projects.

1. Collection of problems—articulating

mathematics education with industrial practice.
2. Publication of pamphlets in cooperation with industry.
3. Summer Institutes with industry.
4. Use of speakers from industry at our meetings.
5. Establishment of Industrial Fellowships.

It was suggested that Dr. Jones send a *complete list* of proposed projects to Miss Rogers for publication in the Newsletter. Miss Lucy Hall recommended that problem collections already made be sent immediately to Dr. Jones.

Dr. George E. Hawkins, National Council Representative to the A.A.A.S. Cooperative Committee on Science and Mathematics, told of the beginnings of the committee and its subsequent services.

This committee has explored many problems. During World War II the emphasis was on a refresher course in mathematics. The National Council took the lead and published a report on this study, which was distributed throughout the country. Directly following the war, the Post-War Policies Commission was organized with Dr. Raleigh Schorling as chairman. This Commission published much excellent material—including the First and Second Reports; a proposed curriculum of study for teachers of mathematics, prepared by Dr. E. H. C. Hildebrandt; and the Steelman report on manpower.

Problems being currently studied by the Cooperative Committee include:
1. Training and certification of teachers of science and mathematics.
2. Shortage of teachers in colleges and universities.
3. Identification of high-school students with potential for science and mathematics, and provision of opportunities for their development.

Miss Agnes Herbert, Chairman of the Committee on Cooperation with N.E.A., brought a brief message to the delegates. She emphasized four ways in which the Affiliated Groups can be of assistance to the National Council and N.E.A.
1. By stimulating participation of more members in the programs of local groups.
2. By supporting the overseas teacher program.
3. By supporting the N.E.A. building program.
4. By supporting the Centennial Action program.

Dr. Henry Van Engen, Chairman of the National Council Research Committee, reported that while considerable research in mathematics and mathematics education is being carried on, it is difficult to stimulate interest in the results of this research.

At present the Research Committee is compiling a report on the Summary of Research in Algebra. Mr. Phillip Peak is heading this committee. Similar reports will later be made on geometry and arithmetic.

Dr. Van Engen also announced that The National Council of Teachers of Mathematics is giving a $1000 award for the best research study prior to 1955 on the topic "What Goes On Inside Individuals When Learning."

As newly elected Editor of The Mathematics Teacher, Dr. Van Engen requested suggestions as to articles and information in The Mathematics Teacher which would prove most interesting and helpful to the teacher of mathematics.

Dr. Van Engen's report was followed by a lively discussion period in which many delegates told of meetings jointly sponsored by their groups with other educational agencies. Persons making these informal reports included:

Miss Lucille Martin—Detroit Mathematics Club.
Dr. Barnett Rich—New York City Association.
Mrs. Mildred Salizman—Indiana Council.
Mr. Lee Dulgar—Men's Math Club, Chicago.
John A. Brown—Wisconsin Mathematics Council.
Jackson B. Adkins—New England Association.
Mr. Oscar Schaaf—Ohio Mathematics Council.
H. W. Charlesworth—Colorado Council.
Miss Helen Graham—Arkansas Council.
Miss Lucy A. Hall—Wichita Math. Association.
Miss Joyce Benbrook—Texas Council.

Dr. Hubert B. Risinger, Chairman of the Speakers Bureau, asked members to suggest topics and speakers. A generous list had already been given. It has been found that there is a better attendance and programs are better since these suggestions have been followed. It is interesting to note a change in the purpose of the Speakers Bureau. It seems to have taken over the problem of learning what topics are of interest in different localities. An experiment is now being made in the Northeastern Area—"Are Topics of Interest in the Northeast of General, Local, and National Interest?"

Miss Madeline D. Messner reported the Travelling Exhibit on display at the convention and invited everyone to examine it. It is comprised mainly of secondary-school materials. It is available for use to anyone interested, the only expense being express charges to the school of the user.

Dr. Mayor suggested that it be sent to the N.E.A. Convention at Miami Beach—expenses paid by The National Council of Teachers of Mathematics.

M. H. Ahrendt discussed the need for closer cooperation between the State Representatives and the Affiliated Groups. He suggested clarification of two problems:
1. Method of selection of State Representatives.
2. Ways and means of bringing about closer cooperation with State Groups.

Mr. Ahrendt recommended that the State

Representatives be appointed before the Fall of 1953 so that they would be ready to start work at the beginning of the school year.

Dr. Daniel Lloyd, Chairman of the Agenda Planning Committee, emphasized the importance of delegates "following through" by reporting to the Planning Committee their reactions to the Assembly program. Favorable comments are always most welcome. Specific suggestions for change and improvement are equally valuable since their wise activation tends to strengthen future programs and procedures.

Existing time conflicts with other important parts of the convention program were noted with great concern. Two suggestions were made for correcting this situation:

1. The first session of the Assembly be scheduled a day earlier than at present.
2. As many Affiliated Groups as possible provide both a delegate and an alternate so that each Group be represented at all deliberations of the Assembly.

Miss Rogers presented the annual Treasurer's report showing a balance on hand of $52.14. The report was accepted.

FINANCIAL STATEMENT

Committee on Affiliated Groups

Balance on hand, April 15, 1952		$ 40.35
Cash receipts		
Dues paid by Affiliated Groups		161.00
Total receipts		$201.35
Disbursements		
Bank service charge	$ 2.00	
Printing 500 affiliation renewal forms	18.00	
Clerical help and postage Chairman of committee on Affiliated Groups	34.97	
Newsletters and postage by regional representatives	41.29	
Speakers' bureau	23.09	
Travelling exhibit	29.86	
Total disbursements		$149.21
Balance, April 8, 1953		$ 52.14

Upon motion the meeting was adjourned.

Respectfully submitted,
DOROTHY SWARD
JEANNETTE GARRETT
Secretaries of the Delegate Assembly

REGISTRATIONS AT THIRTY-FIRST ANNUAL MEETING

Atlantic City, New Jersey
April 8-11, 1953

State	*Members*	*Non-members*	*Total*
Alabama	3		3
Arizona	1		1
Arkansas	1		1
California	4		4
Colorado	1		1
Connecticut	4		4
Delaware	9		9
District of Columbia	26	1	27
Florida	8		8
Georgia	4		4
Illinois	24	2	26
Indiana	10		10
Iowa	4		4
Kansas	1		1
Louisiana	5		5
Maine	1		1
Maryland	27	4	31
Massachusetts	15		15
Michigan	13	2	15
Minnesota	4		4
Mississippi	1		1
Missouri	1		1
New Hampshire	2		2
New Jersey	105	71	176
New York	89	20	109
North Carolina	3	1	4
Ohio	26	5	31
Oklahoma	1		1
Pennsylvania	52	22	74
Rhode Island	1		1
Tennessee	3		3
Texas	4		4
Vermont	3		3
Virginia	16	3	19
Washington	1		1
West Virginia	2	1	3
Wisconsin	9		9
Canada	2		2
Japan	1		1
Totals	487	132	619

The above count is based upon the opinion expressed by the Board of Directors at the Exeter Meeting that the registration report should include only "those who pay the registration fee and attend meetings."

The President's Page

Atlantic City Meeting

At the thirty-first annual meeting in Atlantic City and in the May number of The Mathematics Teacher, announcement was made by the Board of Directors of the appointment of Henry Van Engen, Head of the Department of Mathematics, Iowa State Teachers College, as editor of The Mathematics Teacher for a three-year term, beginning June 15, 1953. The Board believes that we are fortunate in being able to secure a man of Mr. Van Engen's ability for this very important assignment, probably the most important position in our organization. I know that I can express to the new editor, for all members of the National Council, our official congratulations and our very best wishes for great success.

Members of the National Council will be happy to know that E. H. C. Hildebrandt was invited by the Board to serve for a second term of three years and that the appointment of his successor was made only after Mr. Hildebrandt had indicated that his other professional responsibilities made it impossible for him to carry these heavy responsibilities for another term. Not only for Mr. Hildebrandt's fine work as editor but also for his unselfish and devoted service to the National Council as president and Board member are we in great debt to him. Mrs. Marie Wilcox spoke for the Board of Directors in paying a special tribute to him at the banquet in Atlantic City, and the whole Council in the Annual Business Meeting in Atlantic City passed a resolution, expressing our appreciation for his "inspirational leadership and his untiring effort" as editor.

Mr. Hildebrandt was elected to the Board of Directors of the National Council in the spring of 1944, his first year in his present position in the Departments of Mathematics and Education at Northwestern University. In 1947 he was elected second vice-president and in the following spring of 1948, president of the National Council. In 1950 Mr. Hildebrandt was named the first editor of The Mathematics Teacher under the new provision in the Constitution for appointment of an editor for a term of three years.

In the nine years from 1944 to 1953 this man gave most generously of his time and talents to many important activities of our organization. Typical of his leadership qualities, his final report to the Board as editor contained a long list of excellent proposals on present responsibilities and future possibilities for development of the National Council. At all times in his work for our organization, he has demonstrated not only tremendous energy and devotion but real insight and keen imagination, qualities essential to the worth and growth of a professional organization.

John R. Mayor, *President*

A New Publication of the National Council of Teachers of Mathematics. At the Atlantic City meeting of the National Council of Teachers of Mathematics, the Board approved the recommendation of the Committee on an Arithmetic Journal that the National Council establish a journal to be known as *The Arithmetic Teacher*. This journal is to be devoted to the interests of the teacher of mathematics in the elementary school (kindergarten through grade 8). In general *The Arithmetic Teacher* will print articles on the newer methods in arithmetic as well as teaching hints of a very practical nature. The Board hopes that a goodly portion of the magazine will be devoted to publishing materials that the teacher can put to immediate use in the classroom.

The subscription rates for *The Arithmetic Teacher* will be $1.50 per year. Combination rates will be announced. All members of the National Council are urged to make known the Council's intent to publish this new and practical magazine. The first issue? January 1954.

HISTORICALLY SPEAKING, - -

Edited by PHILLIP S. JONES
University of Michigan, Ann Arbor, Michigan

Angular Measure—Enough of Its History to Improve Its Teaching

This note is directed toward two complaints heard recently: (1) that the study of the history of mathematics has not significantly improved the teaching of mathematics in spite of having been recommended or required in teacher training programs for years, and (2) that angular measure, radians and mils in particular, is hard to teach, not meaningful, and should be omitted on the secondary level at least.

The purpose of this note is to show that there are so many interesting and important, pure and applied mathematical notions related to angular measure through the history of its development that this topic taught with these emphases could justify itself solely on the basis of these "incidental learnings," were this necessary.

At the opening of World War II the mil was the one specific topic unanimously seized upon by the committees and textbook authors as mathematics' immediate contribution to the war effort.[1] At that time, however, the real mathematical thinking involved in its development was rarely mentioned and it has now, pedagogically, largely returned to the limbo from whence it came. Although one could argue for its return on the basis of our present world-wide unrest, I choose to argue for a modified reemphasis upon it because of the improvement in students' understanding of mathematics which can result. To develop these ideas, let's turn to the history of angular measure and ask of it as we go the essential question, "Why?"

The system of angular measure taught first, and probably the only system taught with any degree of thoroughness, is the only system for which there is no *logical* justification. Why are there 360 degrees in a complete revolution? There are no reasons for this except historical ones. Even these reasons must eventually give way to hypotheses and these to a mere "because" as is always the case when we continue to ask "why?" and "where?" at each step of an investigation backward into either time or logic. Let's review the story of 360 degrees briefly.

The ancient Babylonians, having settled down (3000–4000 B.C.) to drain marshes, cultivate fields, build cities, and exchange goods, found an interest in astronomy for its own sake, for its relation to religious concepts, and for its connections with the calendar, the seasons, and planting time. They also developed a number system based on 60, using the place value idea for fractions as well as for whole numbers. (The idea of a decimal point and positions to its right representing tens, hundreds, etc., did not enter our Hindu-Arabic number system until about 1585—over 4000 years later!) Why the Babylonians chose 60 no one knows, though there are many interesting

[1] R. S. Burington, "The Mil as an Angular Unit and Its Importance to the Army." *American Mathematical Monthly,* 48 (March 1941), p. 188 ff.; *National Mathematics Magazine,* 15 (May 1941), p. 400 ff.; THE MATHEMATICS TEACHER, XXXIV (May 1941), p. 211 ff. This article contains suggestions for problems and activities.

W. L. Hart, "On Education for Service." *American Mathematical Monthly,* 48 (June–July 1941), p. 353 ff.

theories.[2] It might even be that the use of 60 followed from the easy subdivision of a circle into six equal parts using its radius as a chord. Perhaps the original source of the 60 was as $\frac{1}{6}$ of 360. The idea of 360 parts in a circle might have sprung from a slightly erroneous estimation of 360 days in a year. However, it seems likely that the sexagesimal number system preceded the division of the circle into 360 parts; certainly it preceded the subdivision of each part into 60 subparts.[3] In any event, whether 60 or 360 came first, the Babylonians studied astronomy and used a sexagesimal number system in which fractions were written with denominators of 60 and 60×60 using a place value notion much as we write decimal fractions.

Thus when the Greek civilization through trade and conquest partially absorbed the Babylonian culture, it took its sexagesimal fractions along with its astronomy. Hypsicles (c. 180 B.C.) was the first Greek astronomer to divide the circle of the zodiac into 360 parts, following the Chaldeans who had divided it into 12 signs and each sign into 30 (and sometimes 60) parts.[4] Neither Hypsicles nor the Chaldeans used this division for other circles. This generalization is apparently due to the astronomer Hipparchus (c. 150 B.C.)

Ptolemy (c. 125 A.D.), the famous Greek astronomer and geographer, made a general use of sexagesimal fractions in computation of all sorts, not merely in measuring angles. He did this, he said, to avoid the use of "fractions," thus indicating that the complete idea of a fraction as we teach it in our elementary schools was not clear to this Greek genius, but that he did appreciate the efficiency of the place value notion in computations involving them. He used a decimal non-place value arithmetic for integers, however, and did not use sexagesimal fractions for measuring time. This latter idea was introduced by his commentator, Theon of Alexandria (c. 350 A.D.).

The works of the Greek scientists were preserved, studied, translated into Arabic, extended and combined with knowledge from other sources, such as India, by the victorious followers of Mohammed in the years 600–1200 A.D.

Beginning in the eleventh and twelfth centuries, European scholars became aware of the intellectual treasures preserved by the Moslems and finally began to travel and study, particularly in the Moorish universities and courts, centers of learning in Spain. Gerard of Cremona (1114–1187), for example, learned Arabic in Spain and stayed on to translate at least seventy-six books into Latin.

The Babylonian sexagesimal fractions used in the Arabic translations of the Greek of Ptolemy were called by translators "first small parts" for sixtieths, "second small parts" for sixtieths of sixtieths, etc. The first European translations were into Latin which was the international language of scholarship. In Latin these phrases became "pars minuta prima" and "pars minuta secunda" from which in turn we have our words "minutes" and "seconds."

These words now in daily use epitomize, then, a story reaching back to prehistoric times which gives the real reasons for our use of 360 parts in dividing a circle. Note, though, that these reasons exist as a part of history but not as a part of logic or thought. The choice of units and of methods of subdivision is always arbitrary, but there may be thoughtfully de-

[2] F. Thureau-Dangin, "Sketch of the History of the Sexagesimal System." *Osiris*, VII (1939), pp. 95–141.

[3] F. Thureau-Dangin, "La Division du Cercle." *Revue d'Assyriologie et d'Archéologie Orientale*, 25 (1928), pp. 187–188.

In fact there is even some doubt if the Babylonians themselves ever did divide a circle into 360 parts according to R. C. Archibald in *Science*, 71 (Jan. 31, 1930), p. 117. Bosanquet and Sayce determined that Babylonians at various times used divisions into 8, 12, 120, 240, 480 parts.

[4] Ivor Thomas, *Selections Illustrating the History of Greek Mathematics*. (Cambridge: Harvard University Press, 1941), Vol. II, pp. 395–397.

veloped "logical" reasons for selecting as a matter of convenience and simplification a particular unit or subdivision. Such conscious choice did not enter into the setting up of our degree-(hour)-minute-second subdivisions which are, then, the most popular and the least logical systems of angular measure. All other systems and units have been thoughtfully designed and deliberately named: radians, mils, grades, gones, cirs, decimally or centesimally divided degrees, and millicycles.

Here we will discuss only radians and mils, but before going on to them, let's pause to note the "incidental" mathematics implicit in what we have said so far: the nature and role of definition and undefined terms, and of units, the interaction of mathematics with other elements of our culture as they develop, the nature of number systems and of scales of notation, fractions and the varied approaches to them, both logical and historical!

Radians contrast with degrees in their origin. This contrast is especially sharp if one looks at a strictly factual account as found in the standard works on the history of mathematics.[5] Apparently, mathematician Thomas Muir and physicist James T. Thomson independently considered the need for a new angular unit. Later they met and happened to discuss the need and a name for the unit. They settled upon "radian," after having been consulted by Alexander Ellis, as a compound of "radial angle," though Thomson may have been motivated by an analogy with "median." They had debated other possibilities, such as "radial" rather than "radian." The first appearance of this term in print was on an examination paper set by Thomson in 1873. Its introductions to the general reading public seem to have been in Alexander J. Ellis' *Algebra Identified with Geometry* (London: 1874) where he discussed the exponential form for complex numbers, and in the 1879 edition of Wm. Thomson and P. O. Tait's *Treatise on Natural Philosophy*. However, Todhunter's *Plane Trigonometry* (1891) used the term "radial," while in this country Professors Oliver, Wait, and Jones of Cornell University in their duplicated manuscript, *Notes on Trigonometry* (1880), discuss the "expressing" of angles in terms of "π." In their printed second edition of 1884 they still do not use the term "radian," but refer to "π-measure," "circular," or "arcual measure." None of these authors (I have not seen Ellis or Todhunter) explains why he adopted the unit, but their use of it makes it clear that their reasons lay in the resulting simplification of certain mathematical and physical formulas (especially the derivatives and integrals of trigonometric functions, and the expressions for velocities and accelerations in curvilinear motion).

These formulas are exact. The introduction of radian measure so simplifies them that some would call radian measure the "natural" or "intrinsic" angular unit. However, notions relating angular units to approximation procedures existed long before Thomson and Muir. These ideas seem to have been ignored by historians and teachers as well.

Amos Eaton's *Art without Science: or Mensuration, Surveying, and Engineering, Divested of the Speculative Principles and Technical Language of Mathematics*—was published in Albany in 1830 (first edition 1800?)[6] for use at the Rensselaer School at Troy of whose "practical plan of education" he writes, "Common sense being in its favor, no influence can check its prog-

[5] Florian Cajori, *History of Mathematics*. (Macmillan, 1919), p. 484.

Nature, 83 (1910), pp. 156, 217, 459, 460 contains a series of letters by Thomas Muir and James Thomson's son on this topic.

[6] L. C. Karpinski, *Bibliography of Mathematical Works Printed in America through 1850*. (Ann Arbor: The University of Michigan Press, 1940), on p. 131 notes that "no copy of the 1800 edition has been located, nor is the date certain, for it rests upon the author's assertion in the edition of 1830—that he had published 30 years earlier." *See also* L. C. Karpinski, "Supplement—," *Scripta Mathematica*, VIII (Dec. 1941) 235 for a later partial verification.

ress. The *cloister* begins to surrender to the *field*, where things, not words, are studied!" This forerunner of radian measure is discussed under the general heading of "Extemporaneous trigonometry" and in particular with relation to what Eaton calls "formula 57." In surveying a field Eaton needed to determine the angle of error, A, resulting from an error of 3.10 (3 chains and 10 links) in a distance of 57.80 (Fig. 1). An exact formulation would be

$$\text{(1)} \qquad \tan A = \frac{3.10}{57.80}.$$

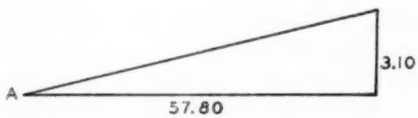


FIG. 1

A table of tangents would be needed to solve this so Eaton uses his "formula 57," which is merely the proportion

$$\text{(2)} \qquad 57.80:57::3.10:A.$$

This gives A to be 3. In accord with his avowed purpose of presenting "art without science" Eaton explains further uses of his rule but never justifies it mathematically. He says, "This formula 57 will give results, which never vary more than four minutes of a degree, through the first ten degrees; though 57.3 is the true measure of radius in degrees, when extended on the periphery of the circle." This latter is of course the clue to the reason behind "formula 57." Rewriting (2) as

$$\frac{A}{57} = \frac{3.10}{57.80}$$

and comparing it with (1) we see that it substitutes $A/57$ for $\tan A$. But $A/57.3$ (Eaton noted that 57.3 is the true measure) is merely A expressed in radian measure. Hence, that which is implicit here is merely that *for small angles the tangent of the angle is approximately equal to its value in radian measure.*

This of course is also true of the sines of small angles. Eaton also uses this second fact implicitly, but to stress the *modern* utility of these ideas I quote a situation and formula from a current surveying text.[7] Distances in the field are often measured along sloping ground as $S = AD$ (Fig. 2). In writing up such a survey the horizontal distance AB is desired. If angle A is known and is small enough, surveying texts recommend the formulas $h \approx .0175\,As$ (A assumed measured in degrees) and $C_s \approx (h^2/2s)$. AB is then $S - C_s$. The point here of course is that

$$.0175 \approx \frac{\pi}{180} \approx \frac{1}{57.3},$$

and $.0175A$ is merely A expressed in radians which in turn is approximately $\sin A$, for A small.

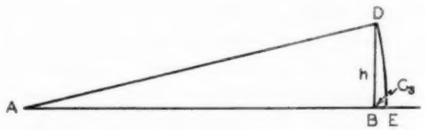


FIG. 2

Although these are approximate formulas and .0175 is an approximate number, their derivation and appropriate use involves good mathematics and functional thinking. If we can, incidental to the teaching of angular measure, also teach functional thinking and the concept that approximations properly treated are not poor mathematics, nor substitutes for mathematics, but good mathematics, then this "incidental learning" may be even more significant than the immediate objective.

[7] Raymond E. Davis and Francis S. Foote, *Surveying Theory and Practice*. (New York: McGraw-Hill Book Co., 1940.)

The use of radian measure for the sines and tangents of small angles depends on the fact that

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1=\lim_{\theta\to 0}\frac{\tan\theta}{\theta}\cdot$$

This can be approached heuristically in several ways: (1) by the use of diagrams showing both the chord of an arc of a circle and the tangent parallel to it along with the central angle, (2) by the use of infinite series, e.g.

$$\sin\theta=\theta-\frac{\theta^3}{3!}+\frac{\theta^5}{5!}-\cdots,$$

(3) by comparing superimposed graphs of $y=\sin\theta$, $y=\tan\theta$, $y=\theta$, (4) by comparing the numerical entries in the angle, sine, and tangent columns of a trigonometric table which shows angles in radians. Whichever of these approaches is used—at whatever level—the others should not be ignored because beginning students are in the category of the cat who "may look at a king." They may be interested, stimulated, motivated by a glance at the marvels the future holds for them, as well as gaining thereby an insight into the interrelatedness of mathematics itself. On the other hand, more advanced students may gain insight and understanding by seeing specializations, applications, elementary illustrations of the more advanced topics which they are studying. In other words, these concepts have teaching utility at levels from tenth-grade geometry (or lower) through the calculus. Further, do not neglect to point out several other places where the approximations $\theta\approx\sin\theta\approx\tan\theta$ are used. Some examples are: the special marks on some slide rules to be used in computing with small angles whose functions are not labeled on the rule,[8] in deriving the formulas for the focal length of lenses in optics (and photography), and for the period of a pendulum (hence every pendulum clock ticks the idea that $\sin\theta\approx\theta$!), the formula for belt friction in machine design,[9] in the analysis of Russell Scott's straight line linkage.[10]

To conclude this part of our survey, let us return to situations using the idea of radian measure in an exact rather than an approximate fashion. In 1722 Robert Smith as editor published a note in Roger Cotes' *Harmonia Mensurum sive Analysis & Synthesis per Rationum & Angulorum Mensuras* on "finding the measure of any angle whatever." This was further expanded by D. C. Walmesley in his *Analyse des mesures des rapports et des angles.*[11]

The "Seconde Parte" of Walmesley's book is titled "De la reduction des integrales aux arcs de cercle." He begins by saying, "The circumference of a circle has been regarded by geometers as the most convenient of all lines and curves for

[8] For example Keuffel and Esser's "Polyphase Duplex Trig" and "Log Duplex Trig" rules have a "minute gauge point" at $(180\times 60)/\pi$ and a "second gauge point" at $(180\times 60\times 60)/\pi$ to be used in finding the sines, tangents, and radian measure of small angles. While we are mentioning slide rules and angular measure note that this company now also makes "deci-trig" rules divided in degrees and tenths of degrees rather than degrees, minutes, seconds for situations and people finding a decimal superior to a sexagesimal subdivision.

[9] See any physics and mechanics texts for these and other illustrations. For example, H. B. Lemon and Michael Ference, Jr., *Analytical Experimental Physics* (Chicago: University of Chicago Press, 1943) p. 483 treats focal length. I. S. and E. S. Sokolnikoff, *Higher Mathematics for Engineers and Physicists* (New York: McGraw-Hill Book Co., 1934) p. 215, and F. B. Seeley and E. Ensign, *Analytical Mechanics for Engineers* (New York: John Wiley & Sons, Inc., 1941), p. 137, deal with belt friction. The latter book derives the formula for the period of a pendulum on p. 346.

[10] Peter Schwamb, Allyne L. Merrill, and Walter H. James, *Elements of Mechanism.* (New York: John Wiley & Sons, Inc., 1938), pp. 123 ff.

[11] Roger Cotes, *Harmonia Mensurum sive Analysis & Synthesis Per Rationum & Angulorum Mensuras Promotae: Accedunt alia opuscula Mathematica.*

D. C. Walmesley, *Analyse des mesures des rapports et des angles on reduction des integrales aux logarithmes et aux arcs de cercle.* (Paris, 1753), p. 52 ff.

measuring angles since the ratios of arcs are the same as those of (central) angles." He then takes up "Problem I. To find the measure of angle AOC (Fig. 3)." He shows that "$\int a^2dx/(a^2+x^2)=$arc AC which is a measure of the angle proposed, AOC." ($x=AD$ in his derivation.)

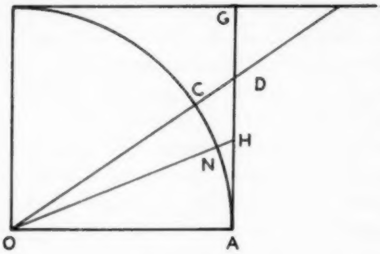


FIG. 3

He then goes on in Corollary I to say, "The size of the arc AC which measures angle AOC is arbitrary. Therefore we must use some 'module' which will determine which one of all the possible arcs in an angle is the one required by the situation in question, since it is the length of the arc and not the size of the angle which determines the value of the integral sought. . . . We imitate the ancient geometers who to fix their ideas of the size of angles found it necessary to divide the circumference of a circle into a number of equal parts without regard to the length of the circumference. . . . Since arcs are in the same ratio as radii and since the radius is the best known line in a circle and enters into all calculations regarding that curve, it is appropriate that it be chosen as the module to which all measures of angles are related."

Corollary II states, "One knows the ratio of the radius to the semicircumference is 1:3, 1415926535 &c., and therefore if R represents the radius we have 3,1415926535 &c.:1::180 dégrés:R and hence $R=57{,}2957795130$ or $57°\ 17'\ 44''$." Walmesley also lets m represent R and $r=1/m=.0174532925$, the conversion factor which we noted earlier in discussing approximations.

Cotes and Walmesley also refer to the idea of "laying off" the radius along an arc and considering the central angle subtended by such an arc. Thus we see that the formal naming of the radian climaxed nearly two hundred years in which the idea had been used in both pure and applied mathematics and in both exact and approximate formulas.

Applications involving the radian measure of angles as exact quantities rather than as approximations are to be found not only in pure mathematics and theoretical mechanics, as suggested earlier, but also in such concrete situations as in optics where the expression $x-\sin x$ has been tabulated, and in the design of gears, especially involute and cycloidal gears.[12]

The mil, too, was deliberately selected and named to serve a particular purpose. Its history seems to be perhaps more recent and certainly more obscure than that of the radian. The word is not even listed in the *New English Dictionary*, the *Encyclopaedia Britannica*, the *Encyclopedia Americana*, or the Swiss *Schweizer Lexekon* (Zurich: 1948, important here because, as noted later the mil may have originated in Switzerland).

Soldiers are often taught that a mil is an angle which subtends one yard at a thousand yards. This is not true. However, this statement is *nearly* true, and this approximation is the key to the chief, and perhaps sole, use of the mil as well as to the reason for its definition.

[12] Earle Buckingham, *Manual of Gear Design* (New York: Machinery, 1935) contains eight place tables of "involute functions" and radians. Werner F. Vogel, *Angular Spacing Tables*, (Detroit: Vinco Corp., 1943) includes central angles for regular polygons from 4 to 200 sides and one table giving angles in radians to ten decimal places. The function $x-\sin x$ for x in radians is important in optics and has been tabulated as have been the functions $\sin 2\pi x$ and $\cos 2\pi x$ for use in the study of crystal structure. Other tables with radian arguments have been prepared for telegraph engineers. See notes by R. C. Archibald in *Mathematical Tables and Other Aids to Computation*, I (Jan. 1943), pp. 14–17.

A few people distinguish between a "true mil" and the mil officially adopted by the United States Army.[13] For such persons a "true mil" is the angle subtended at the center of a circle by an arc equal in length to .001 of the radius. This "one thousandth" accounts for the name of the unit, *mil*, which latter, then, always recalls its relationship to the radian. Further, referring to our earlier discussion of radians, it is easy to see that the sines and tangents of small angles are nearly one one-thousandth of their numerical measure in mils. Thus, in the diagram of an isosceles triangle in Figure 4,

$$\frac{1}{2}S = R \tan \frac{1}{2}A. \tag{3}$$

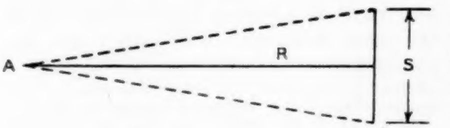


FIG. 4

If A were small and measured in radians, we would have $\tan \frac{1}{2}A \approx \frac{1}{2}A$, and since the mil measure of A (or $\frac{1}{2}A$) is 1000 times its radian measure, we have

$$\tan \frac{1}{2} A \approx \frac{1}{2} \left(\frac{A}{1000}\right), \tag{4}$$

if A is measured in mils. Substituting this in (3) we have $S \approx R \ (A/1000)$. From this it follows that if $A=1$ and $R=1000$, $S=1$, or, a "true mil" is approximately "the angle subtended by a yard at a thousand yards."

There would be 6283.18+ "true mils" in a revolution, an awkward (and irrational —even transcendental) number. This would not be a convenient number for teaching, for use, or for graduating sighting and aiming instruments. Hence, the United States Army arbitrarily defined a mil to be 1/6400 of a circle or revolution. The additional error thus introduced is slight (may I suggest that good, meaningful exercises for students can be based upon comparing the amounts and per cents of error due to these two different definitions for angles, for varied angles and distances). The Army's chief use of the mil is of course in directing artillery fire, determining the original range, and making corrections. Forward observers, often using binoculars with built-in scales for measuring in mils the angles subtended by objects of known or estimated size, have easy arithmetic rules for estimating distances (ranges) and angles.

According to the book *Field Artillery—Basic*, the mil was first introduced by the Swiss army's artillery in 1864. It was adopted by the French in 1879 and by the United States Army in 1900.[14] In at least one branch of the United States Army there has been a trend away from its use,[15] but there is even a semi-mathematical reason for this, and the mil's continued use in many branches is assured for some time.

As noted earlier these few aspects of the still broader topic of angular measure suggest field work and laboratory activities and such teaching aids as range finders, protractors graduated in mils and radians, tables of trigonometric functions for radian and mil arguments, slide rules with

[13] Col. P. S. Bond, U.S.A., *Military Science and Tactics*. (P. S. Bond Publishing Co., 3rd ed., 1941), p. 247.

Amedeo Agostini, "Le Funzioni Circolari . . . ," *Enciclopedie delle Matematiche Elementari*, Vol. II, Parte 1, p. 546, defines a "millesima" as one thousandth of a radian and a "millisima convenzionali" as 1/6400 of a circle.

[14] *Field Artillery—Basic*. (Military Service Publishing Company, 1943), Vol. I, p. 465.

[15] "As rapidly as funds permit, all sighting and other equipment for seacoast artillery using the mil as the azimuth unit will be replaced with new or modified equipment using degrees and hundredths." *Coast Artillery Field Manual*, FM 4-15 (U. S. War Department, 1940), p. 9.

special markings and decimal as well as sexagesimal subdivisions of the degree.[16] Further, we have seen that teaching angular measure with these emphases on its historical development and uses also involves the teaching of approximate computation—both approximate formulas and approximate numbers—the use of tables, graphs, series, simple trigonometry, ratio and proportion. The lengths of arcs, and linear and angular velocity and acceleration, areas of circular sectors and cones are also closely related topics which may be appropriately introduced.

Thus, though it is not the writer's opinion that either the radian or the mil *per se* is an essential part of general education nor that the latter should ever be taught for mastery, it is his contention that teaching *angular measure* with attention to different units, and to the interrelated stories of their *history* which also involves their logical bases, names, and applications, will produce meaning, understanding, insights, appreciations, and even pleasure. These "incidental learnings" are of real educational significance in their own right in addition to the learning of the mere facts with which they are associated.

[16] Houghton Mifflin Company published a four place table of natural and logarithmic trigonometric functions for the argument in mils (10 mil intervals) "through the courtesy of A. W. Tucker, Princeton University." Tables to five significant figures for every tenth mil were available in September 1943 only to the numbers of the applied Mathematics Panel of the National Defense Research Committee. These were the most extensive with the mil as an argument and were restricted because of their importance in ballistics according to R. C. Archibald in *Mathematical Tables and Other Aids to Computation*, Vol. I, p. 146. Other tables and problems involving mils may be found in War Department publications: *Field Artillery Field Manual*, PM 6-40, p. 53 ff., 92-93, 136-139 (use and problems); *Surveying Tables, Technical Manual*, TM 5-236 (trigonometric and conversion tables). Tables for radian arguments and conversion may be found in the *Mathematical Tables from the Handbook of Chemistry and Physics* (Cleveland: Chemical Rubber Publishing Co.) and others. Especially worth noting is the fact that *Tables of Sines and Cosines for Radian Arguments*, prepared by the Federal Works Project Agency conducted under the sponsorship of the National Bureau of Standards (New York: 1940), was one of the first undertakings of this computing project. This is some measure of the importance of radians.

A protractor graduated in mils is available from the Laboratory Specialties, Inc., 144 South Wabash Street, Wabash, Indiana, but it could easily be an interesting project to construct one in school.

Calling Contributors! It is our belief that the history of mathematics, in addition to being intrinsically interesting, can help to build an understanding of the nature of the subject, its relationships within itself and with other fields, and an insight into both the nature of students' difficulties and the manner in which ideas grow. Such understandings and insights should make one both a better mathematician and a better teacher.

We welcome contributions of all types—anecdotes, stories of the development of ideas, and, particularly, suggestions for using the history of mathematics to improve its teaching. What are your historical materials or historical questions?

PHILLIP S. JONES

Then I have more than impression—it amounts to a certainty—that algebra is made repellent by the unwillingness or inability of teachers to explain why we suddenly start using *a* and *b*, what exponents mean apart from their handling, and how the paradoxical behavior of + and − came into being. There is no sense of history behind the teaching, so the feeling is given that the whole system dropped down ready-made from the skies, to be used only by born jugglers. This is what paralyzes—with few exceptions—the infant, the adolescent, or the adult who is not a juggler himself.

J. Barzun: *Teacher in America*, p. 82

DEVICES FOR A MATHEMATICS LABORATORY

Edited by EMIL J. BERGER
Monroe High School, St. Paul, Minnesota

Anyone who has a learning aid which he would like to share with fellow teachers is invited to send this department a description and drawing for publication. If that seems too time-consuming, simply pack up the device and mail it. We will be glad to originate the necessary drawings and write an appropriate description. All devices submitted will be returned as soon as possible. Send all communications to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

DEVICES FOR TEACHING EQUATION SOLVING

The descriptions, drawings and uses of the devices described in this note are based entirely on those enclosed in a letter to the department editor by Mr. Tokuichi Ohtsuka of Japan. His address is:

Kawasaki Lower High School
Kawasaki-machi, Haibara-gun
Shidzuoka-ken, Japan.

Mr. Ohtsuka writes that he has found this type of teaching aids very helpful in teaching his students how to solve "the fundamental simple equations" by applying the "four well known" operational axioms.

Although his letter includes all of the drawings reproduced here, he very carefully added the note that "more general forms (can) be made (based) on the same idea, (except that) more delicate mechanisms may be required."

The main features of any one of his devices are a "cardboard base plate," and a "flap" cut out and pasted as indicated by the shaded areas in Figures 1–5.

Mr. Ohtsuka's letter includes illustrations for two examples. The first of these involves the solution of the equation $2x=10$. To prepare the device for this example first write $x=10$ on a rectangular cardboard base plate (Fig. 1). Then cut out a flap so that it will have a rectangular indentation and paste the ends of the arms thus formed to the base plate. Write the number 2 on the flap so that it appears as the coefficient of x.

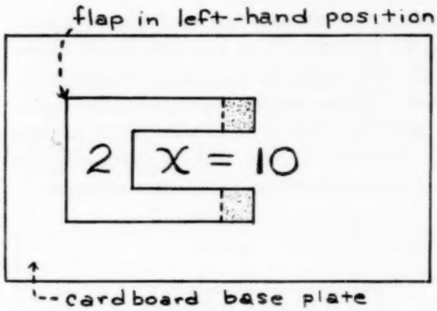


FIG. 1

On the underside of the flap write "x$\frac{1}{2}$." To solve the equation rotate the flap to the right-hand side as in Figure 2. Figure 3 indicates the solution.

Mr. Ohtsuka's second example deals with a device for solving the equation $2x+5=15$. In preparing the device for

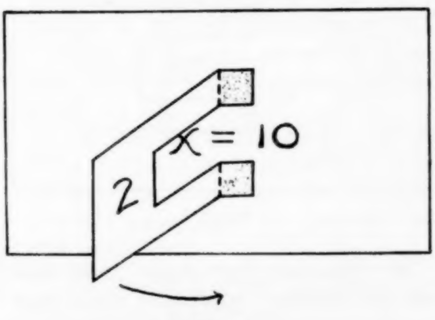


FIG. 2

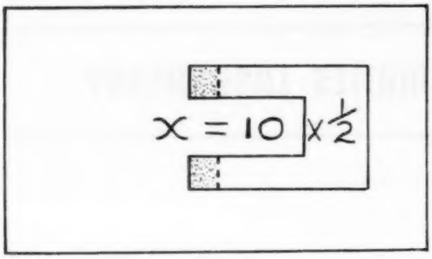


FIG. 3

this equation write $2x=15$ on the base plate and $+5$ on the face of the flap when it is in the left-hand position (Fig. 4). On the back side of the flap write -5. The first step in solving this equation is, of course, to subtract 5 from both members, or to transpose 5 to the right-hand member. This step can be illustrated mechanically by turning the flap over to the right-hand side (Fig. 5). The resulting equation is $2x=15-5$, but this is precisely the situation described in the first example.

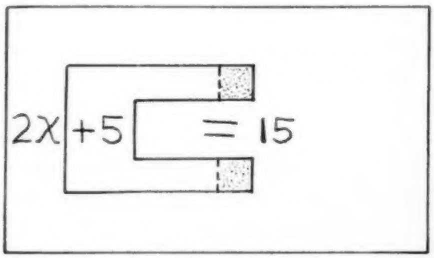


FIG. 4

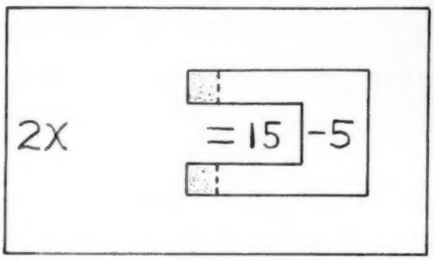


FIG. 5

Mr. Ohtsuka's hint about "more delicate mechanisms" referred to earlier may be called upon at this point to suggest that if the first and second devices are combined —i.e., suppose two flaps are incorporated —then the entire equation could be solved at once simply by performing two operations.

E.J.B.

AN OPTICAL METHOD FOR DEMONSTRATING CONIC SECTIONS

The apparatus illustrated in Figures 6–8 consists essentially of a small, intense source of light mounted at the center of an opaque circular cylinder. A conical beam of light is thereby emitted from each end of the cylinder. These beams, when cut by the plane wall of a classroom, result in an illuminated area the boundary of which is one of the plane sections of a right circular cone. The particular curve obtained depends upon the angle between the axis of the cylinder and the normal to the wall. When this angle is zero a circle is obtained As this angle is increased, ellipses of increasing eccentricity are first obtained (Fig. 6). These are followed in turn by a parabola and hyperbolas (Figs. 7–8). The size of each of these curves may be varied by changing the distance between the model and the wall.

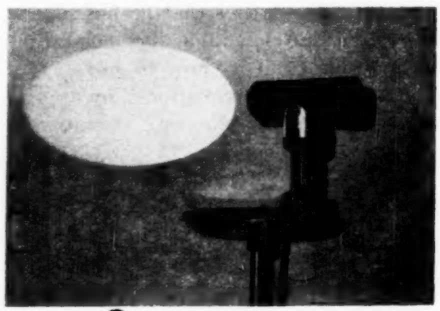


FIG. 6

In the model illustrated, a tee-joint was constructed in the cylinder for convenience in mounting and wiring. The diagram in Figure 9 shows a cross section view of the construction features. The source of light used was a 6-volt fog light bulb. This was wired to a 110–6 volt step-down transformer. The cylinder was lined with black

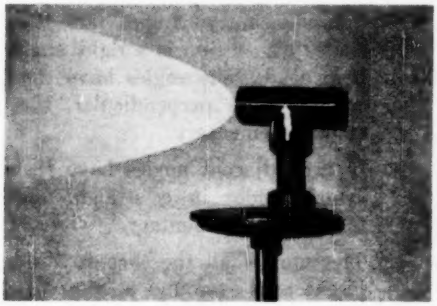


FIG. 7

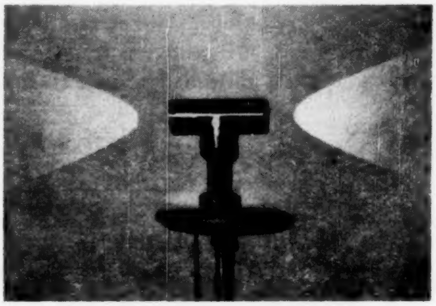


FIG. 8

velvet to minimize reflection of light within the cylinder and thereby increase the sharpness of the shadow boundary. The electric socket and the upper end of the mounting stand were painted black for the same reason. Cooling by natural convection was made possible by boring a large hole vertically through the supporting stand and mounting it on rubber headed tacks as feet.

Simpler forms of this apparatus are possible and are quite satisfactory for use in a classroom which has been partially darkened by drawing the shades. In this connection it is suggested that a three-inch flue pipe lined with black blotter paper be used for the cylinder. An advantage of this simpler form is that a transformer of lower capacity can be employed, thus reducing the expense involved.

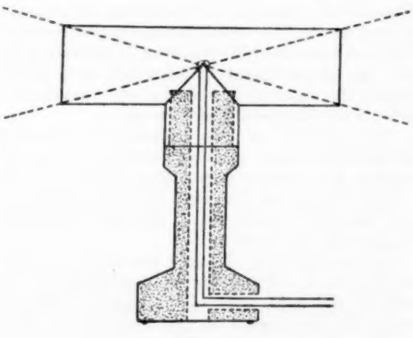


FIG. 9

Editor's Note: At first sight the construction of this type of apparatus may seem to be somewhat complicated, but anyone having mechanical ability and an elementary knowledge of electricity should be able to produce it without difficulty. In fact, a junior student from one of the department editor's classes produced a device based on the information in this article, and it looks almost identical to the one pictured. He used black blotter paper instead of black velvet to line his cylinder, and his "sections" turned out to be quite as sharp as the ones shown in Figures 6–8.

LELAND D. HEMENWAY
Simmons College
Boston, Massachusetts

AN ANGLE DEVICE

An easily constructed device which can be used in a variety of ways is one that may be referred to as an angle device.

The only materials needed to build this device are one stove bolt and four pieces of wood each $1\frac{1}{4}''$ wide, $\frac{1}{8}''$ thick, and having lengths which may vary from $10\frac{1}{2}''$ to $17\frac{1}{2}''$. Sections cut from yardsticks may by used for the wooden parts if desired. As an added feature, each piece may be painted a different color. Assemble the device by putting the stove bolt through holes drilled about $1''$ from one end of each stick (Fig. 10).

With the aid of the device the facts listed below may be illustrated rather readily. The device may also be used to provide opportunity for practice work with these ideas.

1. **An angle is formed by two rays that have the same origin.**

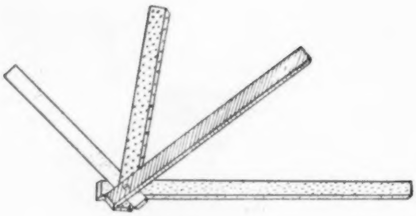


FIG. 10

2. One angle is less than another if it equals a part of the other.
3. A ray bisects an angle if it separates the angle into two equal parts.
4. Adjacent angles are two angles that have the same vertex and a common side between them.
5. A straight angle is an angle whose sides lie in a straight line on opposite sides of its vertex.
6. An acute angle is less than a right angle.
7. An obtuse angle is greater than a right angle and less than a straight angle.
8. The whole angle is greater than any of its parts.
9. The whole angle equals the sum of its parts.
10. Complementary angles are two angles whose sum is a right angle. If two adjacent angles have their exterior sides perpendicular, they are complementary.
11. If two adjacent angles have their exterior sides in a straight line, they are supplementary.
12. The sum of all the successive adjacent angles around a point in a plane equals four right angles or 360 degrees.
13. A perigon is the total angle around a point in a plane.
14. The sum of all the successive adjacent angles around a point on a line on one side of the line is equal to one straight angle or 180 degrees.

If the suggestion made earlier is followed and the four boards painted in different colors, it will be possible for students to indicate by color the particular angle being discussed—i.e., an angle may be referred to as the red-green angle.

SISTER MARY DONALD, I.H.M.
St. Mary High School
Mt. Clemens, Michigan

NEWS NOTES

Marie S. Wilcox, Vice-President of the National Council of Teachers of Mathematics, has been promoted to the headship of the Department of Mathematics at the Thomas Carr Howe High School, Indianapolis, Indiana.

Mrs. Wilcox was formerly a teacher of the George Washington High School, Indianapolis. Her present position will enable her to devote full time to the problems of mathematics in the high school. Her previous position demanded that she spend considerable time on general administrative duties, particularly those pertaining to finances.

Mr. Robert E. K. Rourke, a graduate of Queen's University, Kingston, Ontario, and of the Harvard Graduate School, has been appointed head of the mathematics department at Kent School, Kent, Connecticut.

Since 1947 Mr. Rourke has been headmaster of Pickering College, a preparatory school in Newmarket, Ontario. At Pickering he was appointed head of the mathematics department in 1928 and continued his teaching of mathematics after his appointment to administrative work. During the summer of 1952 he was visiting lecturer at the University of Alberta.

Mr. Rourke is the co-author of several mathematics texts and has written numerous articles for professional publications of the United States and Canada. He is a member of the executive committee of the Canadian Mathematical Congress.

At Harvard Mr. Rourke was twice granted the Shattuck Scholarship in mathematics. He received his A.M. with distinction in 1930 and completed his courses for the doctorate in 1931.

Mr. Rourke and his family will take up residence in Kent on August 1.

The National Council of Teachers of Mathematics

Research Award

of

One Thousand Dollars

The National Council of Teachers of Mathematics Research Award of \$1,000 is available to a doctoral candidate or an individual carrying on independent research in learning problems in the field of mathematics, including arithmetic. The purpose of the Award is to encourage research studies in this area. The Award will be granted on the basis of the best prospectus as judged by a panel chosen by the Board of Directors of the National Council. No Award will be made if in the opinion of the judges no prospectus submitted shows promise of producing significant results in the learning of mathematics.

A prospectus of the research problem must be submitted prior to January 1, 1955, and satisfy certain basic conditions designated by the Board of Directors of The National Council of Teachers of Mathematics. Payment of the Award will be contingent, in part, upon progress made by the proposer in carrying out the research problem.

For further details, write to M. H. Ahrendt, Executive Secretary, The National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington 6, D.C.

I Love to Teach

I do not know that I could make entirely clear to an outsider the pleasure I have in teaching. I'd rather earn my living by teaching than in any other way. In my mind, teaching is not merely a life-work, a profession, an occupation, a struggle. It is a passion. I love to teach. I love to teach as a painter loves to paint, as a musician loves to play, as a singer loves to sing, as a strong man rejoices to run a race. Teaching is an honor, an art so great and so difficult to master that a man or woman can spend a long life at it without realizing much more than his limitations and mistakes and his distance from the ideal. But the main aim of my happy days has been to become a good teacher just as every architect at every professional point strives toward perfection.

—William Lyons Phelps

MATHEMATICAL MISCELLANEA

Edited by

Paul C. Clifford
State Teachers College
Montclair, New Jersey

and

Adrian Struyk
Clifton High School
Clifton, New Jersey

Keep the Score Down, Boys

Our appearance as editors reminds us of the old plea to the substitutes at our alma mater. The first team was pretty good, but there was a lack of reserves on the bench. Sometimes we felt that all the subs did was to keep a team on the field. After reviewing the remarkable job that Phillip Jones did as editor of this column, we feel like the last substitute on the bench, the one who has never been in a game before and whose reaction is, "Who, me?" when the coach beckons.

Come on in, the Water's Fine

Our hope is that we will be able to sit back, put our feet on the desk, and edit. Of course, that means that somehow we have to get some of you potential contributors to send in your pet items.

We are not quite sure what a "miscellanea" consists of. Our dictionary says that it is "a collection of miscellaneous matters"; "miscellaneous" in turn means diverse things having various qualities. Then, as a synonym for "miscellaneous" it gives "indiscriminate." As we see it, if you will only send in enough diverse things of various qualities, we can be indiscriminate in our editing, and we can quote Webster as our authority!

It Seems to Me I've Heard that Song Before

One question that arises is that of originality. We will leave this fascinating question to those poor souls who sponsor Ph.D. dissertations. If an item interests us, and we think it will interest you, that is enough. For example, we submit:

84. Treasure Hunt

This is familiar to those of you who have read *One, Two, Three, . . . Infinity.*[1]

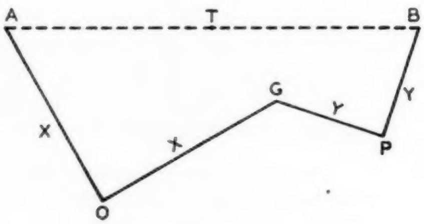


Fig. 1

The treasure is referenced to a gallows G, a pine tree P, and an oak tree O as shown in Figure 1. The instructions are: Point A is at a right angle (counterclockwise) from OG, with distance AO equal to OG. Point B is at a right angle (clockwise) from PG, with distance BP equal to PG. The treasure T is located midway between A and B. The only difficulty is that all trace of the gallows G has disappeared.

Gamow gives an interesting solution using complex numbers. However, this is a relatively easy problem for a plane geometry, trigonometry or analytic geometry class. The trick is to draw the x axis through OP, with the y axis best located as the bisector of OP. T is then located as the point $(O, OP/2)$.

[1] George Gamow, *One, Two, Three, . . . Infinity* (New York: Viking Press, 1948).

What is unusual about the problem, of course, is the fact that the location of T is independent of G. And the question we ask is: Is this one of a large number of such problems? We would be interested in hearing of others like it.

85. Beware the Continued Radical

Harry Schor of Far Rockaway High School, New York, has sent in the following, which is the work of Marc Brown, a student. We give an abbreviated treatment, for we feel that in this, like the preceding item, the unusual aspect is that a student developed this as an original. Using a regular n-sided polygon inscribed in a unit circle, the perimeter is found to be $2n \sin 180/n$. Since the circumference of the circle is 2π, he takes $\pi = \lim_{n\to\infty} n \sin 180/n$ and by repeated use of the half-angle formulas for sine and cosine he evaluates this where n is of the form 2^x. This leads to the following expressions:

$$\sin 180/2^x = \tfrac{1}{2}\sqrt{2_1 - \sqrt{2_2 + \sqrt{2_3 + \cdot\cdot\sqrt{\cdots + \sqrt{2_{x-1}}}}}}$$

$$\cos 180/2^x = \tfrac{1}{2}\sqrt{2_1 + \sqrt{2_2 + \sqrt{2_3 + \sqrt{\cdots + \sqrt{2_{x-1}}}}}}$$

$$\pi = \lim_{x\to\infty} 2^x\sqrt{2_1 - \sqrt{2_2 + \sqrt{2_3 + \cdots \sqrt{2_x}}}}$$

The completed proof by mathematical induction has also been given. Now returning to the question of originality, we ask: Is this original in the sense that a high-school student developed it? Or is it typical of what capable students are doing under good teachers? We hope that you will prove it is the latter by sending in more original work of your students.

86. A Review Lesson on Logarithms

We are intrigued by the suggestion for a review lesson on logarithms, sent in by Norman Anning, formerly Professor of Mathematics at the University of Michigan and now retired in Alhambra, California. He obviously is not retired at all as far as his interest in mathematics is concerned.

Have the typist make for each student a copy of the following table:

Number	*Logarithm*
1	0.0
$a = 1.58489$	0.2
$b = 2.51189$	0.4
$c = 3.98107$	0.6
$d = 6.30957$	0.8
10	1.0

A flock of products, quotients, powers, roots (some), and reciprocals can be found about as fast as the teacher can suggest them. And if harder problems are called for, here are some.

1. Use these numbers in verifying the theorem that $(\log_b a)(\log_a b) = 1$.

2. Show that the product when $(a-1)$ is multiplied by

$$(1+a+b+c+d) \text{ is exactly } 9.$$

If a table twice as long is preferred, the numbers 1.25893, 1.99526, 3.16228, 5.01187, 7.94328, with their logarithms may be suitably inserted in the short table given above.

87. Here We Go 'Round. . . .

Norman Anning sends in some examples of equivalent fractions that are obtained by permuting the integers that appear. As he says, "There is no rule for finding such relations any more than there is a rule for finding agates or truffles. Go where they are likely to be." That reminds us of the famous question "Who wrote . . . what was it?" Here are a few to puzzle over:

$$\frac{243}{324} = \frac{324}{432}$$

$$\frac{44649}{64944} = \frac{64944}{94464}$$

(*Continued on page 443*)

WHAT IS GOING ON IN YOUR SCHOOL?

Edited by

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Wisconsin High School
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and

Houston T. Karnes
Louisiana State University
Baton Rouge, Louisiana

A Workbook for Mathematics III

Compiled by the Janesville Senior-High Mathematics Department

Our course in Mathematics III was organized in 1942 for eleventh-grade students to meet a demand for mathematics of a more practical type than the college preparatory course offered at that time. At the present time, the subject matter for the Mathematics III and Algebra II courses is the same, or nearly so, for the first ten weeks, which is the time allotted to cover the first two units. The workbook is used to provide practice materials for both courses for these two units.

Unit I. Measurement and the Slide Rule

The unit begins with a review of whole numbers and decimals. It includes such topics as construction of the number system to the base ten, exact and approximate numbers, precision computation, and placement of the decimal point by the estimation method in multiplication and division of decimals, in preparation for computation with the slide rule.

The next part of the first unit is Systems of Measurement, which is a review of the English system of weights and measures, an introduction to the metric system with its advantages and its similarity to our number system to the base ten, and conversion from one system to the other. The students find this work helpful to them in their beginning work in physics.

The third part of Unit I is Multiplication and Division on the Slide Rule. The entire work of Unit I is reviewed in this section, use being made of the estimation method for pointing off answers obtained on the slide rule, rounding off numbers, applying the rules for approximate computation, and performing on the slide rule the multiplications and divisions involved in converting from one unit to another.

Unit II. Logarithms and the Slide Rule

This unit begins with a review of the four fundamental operations with signed numbers; the laws of exponents and multiplication, division, powers, and roots, with zero, negative, and fractional exponents, and scientific notation. Then follows work in performing multiplications, divisions, powers, and roots by using tables of powers of two, three, and ten, thus introducing logarithms. The general definition of a logarithm is given, followed by practice in changing from exponential to logarithmic form and vice versa, using all types of exponents. The usual work on computation with logarithms to the base ten is given, including solution of the exponential equation and its application to compound interest and compound discount. The unit also contains further work on the slide rule, such as combination multiplication and division and square root. The slide rule is used for checking the computations with logarithms.

Difficulty in finding practice materials prompted us to make our own work-sheets. As these accumulated, the mere handling of them became somewhat of a burden, hence the workbook which was made entirely for our own use. It contains only those exercises which could not be found in our texts.

Following is a description of them.

1. Eight exercises on changing from one unit to another within the metric system and conversion from one system to the other.
2. Ten exercises developing step by step the understanding and skill for pointing off products in multiplication of whole numbers and decimals, eight exercises for developing understanding and skill in pointing off in division of decimals, and four exercises providing practice in mixed multiplication and division.
3. Twenty-five exercises for learning to use the slide rule, beginning with first division and then multiplication of two-digit numbers, then using three-digit numbers between 1 and 2, next three-digit numbers between 2 and 4, and finally three-digit numbers from 4 on. The next exercises provide practice on mixed multiplication and division. There are exercises on finding the product of three or more numbers, on combined multiplication and division, on square root, and solving proportions.
4. Five exercises on performing computations with exponents. The first one is on evaluating numbers with zero, negative, and fractional exponents. The second exercise is a table of powers of 2 and 3. Exercise three gives the value of 2 with exponent $\frac{1}{2}$, 2 with exponent $\frac{1}{3}$, and also 3 and 10 with the exponents $\frac{1}{2}$ and $\frac{1}{3}$ from which the value of 2 with exponents $\frac{3}{2}$, $\frac{5}{2}$, and $-\frac{1}{2}$, etc., are to be found. Exercise four makes use of the answers in exercise three for finding products, quotients, powers and roots, thus providing practice with both positive and negative fractional exponents. Exercise five develops an understanding of finding the characteristic of the logarithm of a number, given the characteristic of its logarithm, so that when the transition is made from the exponential form to the logarithmic form no rule need be given. This exercise also affords practice in finding products, quotients, powers, roots, by expressing numbers as powers of ten.
5. Four exercises on logarithms. The first one gives the general definition of a logarithm, shows how to change an equation from logarithmic to exponential form and vice versa, and provides for practice in the use of all types of exponents. The next two exercises are on interpolation. A reference sheet is given showing a convenient arrangement of work when computing with logarithms, followed by practice exercises.

Marjorie Davis, Chairman
Mathematics Department
Janesville Junior High School
Janesville, Wisconsin

Experimental Work in Plane Geometry

West Allis is an industrial city. There are continuous demands for the school system to supply workers with a mathematics background. Since algebra and geometry are not required courses in the local high schools, students enrolled in such courses are there because they elected to be there.

In the summer of 1952, West Allis Central High School purchased adjustable triangles, extensible triangles, quadrilateral devices, circle devices, parallel line devices, and criteria quadrilaterals for experimental use in a plane geometry class. The equipment is modern looking and pleasant to handle. The room where the equipment is used is ideally furnished with formica-topped tables.

The adjustable triangle is aluminum. One side is red, another is blue, and the third is gold. The sides are 35 centimeters long and are constructed so that a great variety of sizes and shapes of triangles can be obtained. Protractors are mounted rigidly at the vertices and on the three movable fixtures at the sides. Colored elastic cords can be fastened to the vertices and the movable points. A triangle is provided for each member of the class. The attached work sheet is one example of the many relationships that can be investigated with the adjustable triangle. Other relationships follow:

1. Congruence.
2. Similarity.
3. Sum of the angles of a triangle.
4. Exterior angle of a triangle.
5. 30°, 60° right triangle.
6. 45°, 45° right triangle.
7. Angle relations in a triangle having three equal sides.
8. Side relations in a triangle having two or three unequal sides.
9. Side relations in a triangle having two or three unequal angles.
10. Side relations in a triangle having two or three equal angles.
11. Relationships involving altitudes to equal sides of a triangle.
12. Point intersection of angle bisectors of a triangle.
13. Relationships involving medians of a triangle.

14. Division of two sides of a triangle by a line parallel to the third side and the converse relationship.

It was found that the teaching of geometry was improved greatly by doing, as well as by seeing and hearing. The adjustable triangle put into the hands of each student enables him to enter actively into a learning situation. Hearing, sight, and muscular co-ordination unite to create new ideas and provide a way to discover, interpret, and formulate problems inductively. Induction and deduction complement each other as each is necessary for complete understanding. An attempt was made to develop skills in working with the relationships of variables.

For example:

What combination of conditions will bring about certain desired results?

Knowing what is wanted, what must be done to find it?

Having in mind certain changes in the result, what changes must be made in the conditions?

What are the consequences from a given set of conditions?

Knowing what the conditions are, what do they mean and what can be done with them?

If a change is made in one or more of a set of conditions, how will the result be affected?

In April each member of the four geometry classes chose a geometric idea as the basis for constructing at home a movable model. Materials used were hardwood, plywood, wallboard, linoleum, heavy cardboard, paint, round elastic, round-headed screws, push pins, protractors, and rulers. Most finished models were 18″ by 24″. The project covered a three-weeks period and models started coming in at the end of one week. For the following two weeks, each morning was like Christmas morning for students and teacher alike. The continued anticipation and enthusiasm helped to quiet the qualms of one who was responsible for such an unheard of assignment in plane geometry.

A geometry classroom equipped with learning aids, models, and instruments provides a practical setting for student discovery of quantitative and spatial relationship. Problems based on student discovery have more educational value than those assigned from a text. Prestated theorems and prefabricated exercises are often artificial, superficial, and dull.

Claire Helstern
West Allis Central High School
West Allis, Wisconsin

Activities Which Create Interest in a Mathematics Class

All teachers are faced with the challenging problem of maintaining a high level of interest in the classroom. This seems particularly difficult for some teachers of mathematics. However, there are many devices which an alert teacher can employ, activities and techniques which assist in keeping the class interest high and enrich the everyday learning experiences in the classroom. The writer is happy to share the following practical, specific experiences with the hope that they may be helpful and suggest similar activities.

(1) The use of statistics of local community.

When studying per cents of increase and decrease, a ninth-grade arithmetic class noticed an article in the local newspaper which discussed the increase in population of the communities which comprised their school district. The class checked the percentages claimed in the article and verified them. It was then decided to make a map, suitable for the mathematics classroom wall, which would show the growth figures. Pupils were divided, according to their own choice and ability, into committees. Each committee was assigned a definite part of the work in order to carry out the proposed project.

One group secured a large map of the communities, which they traced on a piece of plywood. Another group wood-burned the outline of the communities and made a

border around the map. A third group painted the communities different colors. A fourth group secured packets of gold letters, which were sold in the school bookstore, and pasted these on the map, indicating the name of the community and the per cent of growth for the past ten years. The class discussed a suitable title, and lettered it on the map. Each pupil autographed the back of the map, the date of completion was printed on it, and finally the completed map was hung on the wall at the back of the room, where it makes a colorful, interesting, and appropriate decorative feature.

(2) Bulletin board decoration.

A seventh-grade arithmetic class was studying the unit on geometry. Part of this unit was learning to recognize the various geometric shapes and to use them in decorative ways, such as making geometric designs. Curve-stitching had been introduced as one way of arriving at satisfactory designs. Since it was in early December that this unit was being studied, the teacher suggested that pupils who were interested might like to curve-stitch designs which could be used as ornaments for a large white velour paper Christmas tree on the bulletin board. There was much eagerness on the part of the pupils to bring in designs which they had made at home. There was a great deal of favorable comment by pupils, parents, and teachers on this novel decorative Christmas tree. The bright colors of the thread designs really gave the tree a festive look.

(3) Geometric Christmas tree project.

Another seventh-grade class in arithmetic designed and made an appropriate geometric Christmas tree for the window of the classroom door. A large piece of tagboard was secured and cut to size. The tree was then drawn, made of three overlapping equilateral triangles, each somewhat larger than the one above. Duplicate triangles were cut out of green construction paper. The various geometric shapes were drawn on the tagboard tree to look like Christmas tree trimmings, and these were then cut out. The green construction-paper triangles were fitted over the duplicate tagboard triangles, and the cut-out geometric shapes traced on them. These were also cut out. Pupils brought colored cellophane, which was then fastened to the tagboard with a paper stapler. The green construction-paper triangles were stapled into place. The background was made of aluminum foil which had been crumpled and then straightened out to give a textured appearance. A border of red construction paper was stapled on and the whole finished piece fastened to the glass window of the door. The decorative tree was made so that when observed from either side it looked the same. Light filtering through the colored cellophane gave the appearance of illumination. All agreed that the classroom door was very attractive and appropriately decorated.

(4) Assignment sheets.

Each Monday, one of our mathematics teachers prepared hectographed assignment sheets for his classes. At the top of this sheet, the heading tells the week and date, and along the left side the days of the week are printed. The teacher discusses the assignments and writes them on the board, and each pupil fills in his assignment sheet. This is helpful to parents as well as to the pupil and the teacher. The bottom portion of the sheet is reserved for mathematical puzzles, several of which are given each week, varying in difficulty. The answers to the previous week's puzzles are printed on the next assignment sheet. Whenever pupils desire, a discussion of the puzzles is given in class. Parents are often as interested in the puzzles as the pupils. Sometimes the teacher gives clues to assist the pupils in getting started correctly. At times cartoon sketches, such as the donkey and the elephant for election week, are included. There has been much favorable comment from parents.

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APPLICATIONS

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Ar. 31 Gr. 6–12 *Tire Sizes*

For modern automobiles the tire sizes range from 5.50–15 to 8.20–15. What do these numbers mean? If they are true sizes, there should be units of measurement included or implied. What are the units? Does a person have to measure his old tires, or have them measured, when they need to be replaced? Perhaps these numbers are only for classification or cataloging and are not true sizes or measurements. Is that true? What about the point in the number before the dash? Is it a decimal point?

These questions have come to the mind of this writer on several occasions and it took some investigating to find reliable answers, for familiarity with this particular use of number is extremely varied. Some persons just do not know; others feel that the numbers represent a diameter, a tread width, a tread depth, a tire thickness, and so on. Can the readers of this department explain how a tire size is related to the tire itself? Or better, can the students in their classes describe this relationship? Since cars and most things connected with cars are of great interest to most teen-age youngsters, it is appropriate to include this use of number in a school mathematics class.

Using the size 6.00–15 to illustrate, the first number in a tire size shows that the tire is 6.00 inches thick at its thickest part. The second number shows that the inner diameter of the tire is 15 inches. In many trades, the context of the situation makes clear what the units of a given measurement are. So it is in the case of the tire trade. In fact, when tire sizes are spoken, the decimal point and the dash are usually ignored. Thus, 6.00–15 is spoken "six hundred fifteen" and 7.60–16 is spoken "seven sixty sixteen." Not even in price lists are the units of measurement in tire sizes given. Thus it is not unusual that some persons regard tire sizes as catalog numbers, similar to library index numbers.

When tire sizes are seen in their true perspective as measurement numbers, there appears to be a difference in the precision of the two parts of a tire size. One is tempted to infer that the thickness is measured to the nearest one-hundredth of an inch while the diameter is measured only to the nearest inch. This department would appreciate hearing from its readers on this point. *Is* the thickness measured to greater precision? If so, why? If it is not, why is the first number *expressed* to greater precision than the second?

Ranges of tire sizes of representative types of tires are given below. Notice what appears to be less precision in the measurement of the thickness of rear tires of tractors.

Automobiles: 4.40–21 to 8.20–15
Trucks: 7.00–15 to 14.00–24
Earth movers: 11.00–20 to 30.00–33
Tractors: Rear: 10–38 to 18–26
Front: 3.00–12 to 9.00–10
Garden tractors: 4.00–8 to 7.50–22
Motorcycles: 3.25–18 to 5.00–16

Here are some questions which might arouse the interest of some students:

1. Make a list of the tire sizes for automobiles. If the list is arranged in order of the increasing cost of the automobile, do the tire sizes increase also? If the list is made according to the increasing length of the wheelbase, do the tire sizes increase also?
2. If your bicycle tires were given sizes

in the same manner as other tires, what would the size be?

3. If you classified the tires on your smaller brother's (or sister's) tricycle by this method, what would the sizes be?
4. Without asking the size or looking at the tire size marked on the tire, measure the rear tire of a farm tractor and see if you can determine its correct size.

The problem of making the first of the two measurements in a tire size affords an excellent opportunity to see the need for a measuring instrument like the calipers.

Ar. 32 Gr. 6–12 *Pharmaceutical Arithmetic in the Home*

There is no question that in pharmaceutical work there are significant applications of mathematics through the use of different systems of measure (English and metric), units of measure rarely used by the layman (grain, gram, cubic centimeter, minim, dram), and very small quantities (a hundredth of one per cent, a thousandth of a milligram).

If one tries to read the information given on the label of a bottle of vitamin drops given to children or of capsules taken by adults, one begins to wonder whether or not the average layman should not know some of the quantitative vocabulary at least, if not also some of the arithmetic processes of ratio and proportion used daily by a pharmacist.

The following can be called problem situations which would face a father, mother, brother, sister, baby sitter, or anyone who might, from necessity or curiosity, be compelled to read and to understand labels on containers of vitamins.

1. On the label of a bottle of *Unicap* vitamin capsules, the following data are given:

1 capsule contains:
5000 units of vitamin A ($1\frac{1}{4}$ MDAR)
500 units of vitamin D ($1\frac{1}{4}$ MDAR)
37.5 mg. of vitamin C ($1\frac{1}{4}$ MDAR)
2.5 mg. of vitamin B_2 ($1\frac{1}{4}$ MDAR)
2.5 mg. of vitamin B_1 ($2\frac{1}{2}$ MDAR)
(MDAR: Minimum Daily Adult Requirement)

What number of units makes the MDAR for vitamin A? for D? for C? for B_2? for B_1?

2. Are there standards set up for minimum daily requirements for vitamins for each drug company to follow? Looking at another product helps answer this. *Multi-Vi* vitamin drops are produced by a different company than the one producing *Unicap*. Below are the data from a bottle of *Multi-Vi* drops:

0.6 cc. contains:
5000 units of A (125% MDAR)
1000 units of D (250% MDAR)
1 mg. of B_1 (100% MDAR)
0.4 mg. of B_2 (20% MDAR)
50 mg. of C (167% MDAR)

Find the MDAR for each of the vitamins above and see if they agree with the requirements found in situation No. 1.

3. *Tri-Vi-Sol* is a solution of vitamins prepared primarily for infants up to one year of age. The directions state:

0.6 cc. supplies:
5000 units of A (3.3 MDIR)
1000 units of D (2.5 MDIR)
50 mg. of C (5 MDIR)

Compare the MDIR with the MDAR for vitamins A, D, and C. Do these figures show any definite relationship between the requirements for adults and for infants? For example, do adults need twice as much of each vitamin as do infants?

4. One ounce of *Gerber's Cereal Food* supplies 320% MDIR for vitamin B_1 and 120% MDIR for vitamin B_2. The number of units of these vitamins is not given on the label.

How many units of B_1 are there in a 1-oz. serving of this cereal?

What would be the weight of a serving of this cereal which would provide the MDIR for vitamin B_1? for B_2?

(Continued on page 447)

REFERENCES FOR MATHEMATICS TEACHERS

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Map Projections and Cartography

"A map is, in its primary conception, a conventionalized picture of the earth's pattern as seen from above . . . Some maps are abstracted and conventionalized to such a degree that the original notion of a picture is hardly recognizable."—Erwin Raisz

"Every map projection is an abstraction in which certain qualities of truthful representation are of necessity sacrificed in order to preserve others more relevant for the particular purpose of the particular map."—J. Q. Stewart

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Mathematical Miscellanea

(*Continued from page 433*)

For the first one, Mr. Anning suggests that we simplify the periodic decimal fractions $0.\dot{2}4\dot{3}$, $0.\dot{3}2\dot{4}$, $0.\dot{4}3\dot{2}$, and try to discover the reason and the source of the relationship. We suggest that $4\cdot 8=32$, $3\cdot 8=24$, is a good start, since the 2, 3, 4 appear in each. Try replacing the 8 by 81 or 108.

For the second one, Mr. Anning suggests that we consider $a/271=b/99999$. Well, that's just what we have done—considered it. Maybe by next month we will have done our homework.

AIDS TO TEACHING

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BOOKLETS

B. 164—Days for Dates

Walter R. Cuthbert, 1010 North Garfield Avenue, Alhambra, California.
Booklet; 32 pages; 6″×9″; $1.00.

Description: This publication presents a unique method of ascertaining on what day of the week a date in a given month in a given year has occurred or will occur. Simple tables and sample computations are used to show a method which the author claims can be learned by a child of 12 in one hour's instruction. The method involves memorizing a few key years, coding the months and reducing the day of the month by casting out sevens. The booklet includes practice exercises with answers, memory aids, and a simplified perpetual calendar.

Appraisal: This is a booklet that will furnish an appropriate recreational activity or supplementary assignment for junior or senior high-school students. For those who are interested, it will furnish a mental calendar that will always be available to determine any date in the past or future.

B. 165—Your Budget

B. 166—Your Food Dollar

B. 167—Your Clothing Dollar

B. 168—Your Home Furnishing Dollar

B. 169—Your Shelter Dollar

B. 170—Your Recreation Dollar

B. 171—Your Shopping Dollar

B. 172—Your Health Dollar

B. 173—Children's Spending

Household Finance Corporation, 919 North Michigan Avenue, Chicago, Illinois.
Booklets; 24 to 36 pages; 6″×9″; $0.10 each.

Description: These money management booklets deal with the management of family income, how to plan expenditures to best advantage, and how to buy wisely. Frequently, forms for keeping accounts are included. Each booklet usually begins by outlining basic needs in terms of the products being discussed. Information is included on how to judge the quality of products—the standards, characteristics or units used. Emphasis is placed on careful planning of purchases according to personal needs and resources. Specific suggestions are given on how to shop in order to get your money's worth.

Appraisal: The amount of mathematical material in these booklets is limited. However, they will be very useful for supplementary reading in general mathematics, home economics, business arithmetic, or consumer education courses. The booklets are well written in a concise outline form with frequent illustrations. They were prepared in consultation with many well-known authorities in the field. The amount of advertising by the publisher is limited.

EQUIPMENT

E. 127—Self-Teaching Flashers Arithmetic Games. Set A: Addition-Subtraction.

E. 128—Self-Teaching Flashers Arithmetic Games. Set D: Division-Multiplication.

Self-Teaching Flashers, 4402 S. 54th St., Lincoln 6, Nebraska.

Sets of 81 cards, each approx. $2\frac{3}{4}'' \times 4\frac{1}{2}''$; \$1.00 per set; 1945 and 1952

Description: The cards in Set A cover the 81 basic A- and S-combinations (excluding the zero-combinations). Each card has a basic A-combination on one side and a related S-combination on the reverse side: e.g., $\begin{array}{r}3\\+7\\\hline\end{array}$ and $\begin{array}{r}10\\-7\\\hline\end{array}$. The combinations are so arranged that the answer to the A-combination on one side is the "top number" of the S-combination on the reverse side; and conversely, the answer to the S-combination on one side of any card is the "top number" of the A-combination on the reverse side. In like manner, the cards in Set D cover the 81 basic M- and D-combinations. Similar to Set A, each card has a basic M-combination on one side and a related D-combination on the reverse side: e.g., $\begin{array}{r}7\\\times 4\\\hline\end{array}$ and $\begin{array}{r}28\\\div 4\\\hline\end{array}$ (the algorism for the D-combination being as just shown). Again, the combinations are so arranged that the answer to the M-combination on one side of a card is the "top number" of the D-combination on the reverse side, and vice versa. The playing of the game with either Set A or Set D cards follows fairly typical flash-card routine and procedure.

Appraisal: The reviewer is concerned seriously about certain "features" of the cards and about certain claimed "advantages" for their use. Relative to each set it is stated: "Each room should have from five to ten sets thus increasing skills in like proportion." Certainly clear-thinking persons are not blind to the grossly misleading implications of such a statement! Or consider the following bit of erroneous information: "Since the top number is the answer to the combination on the other side of the card the pupil learns the relation by simply turning the card to see the answer each time. Elementary supervisors appreciate this feature because it is the proper method of teaching number relations." *This is* NOT *the proper method of teaching* MEANINGFUL *number relations.* Certainly there should be no need to expand upon this obvious fact. Notice that the combination $\begin{array}{r}7\\+3\\\hline\end{array}$ is just as related mathematically to the combination $\begin{array}{r}10\\-7\\\hline\end{array}$ as is the combination $\begin{array}{r}3\\+7\\\hline\end{array}$. This fact is completely disregarded in the Self-Teaching Flashers method of "teaching relationships." It is disregarded of necessity because such a pairing of combinations would not fit the scheme of mechanical and arbitrary association employed in constructing the cards. The final absurd way in which the authors have become slaves to their own system is seen in the algorism they have been forced to employ with the division combinations: $\begin{array}{r}28\\\div 4\\\hline\end{array}$. Since neither conventional algorism, $4\overline{)28}$ or $28 \div 4$, could be used with their arbitrary system of association, there was but one alternative: keep the system, but increase greatly the probability of confusing children by presenting a third algorism. Acceptance of this alternative does not seem consistent with recognized principles of meaningful arithmetic instruction. (Reviewed by J. FRED WEAVER, Boston University.)

*E. 129—*MATHEMATICS TEACHER *File*

James F. Ulrich, 3442 Mackinaw Street, Saginaw, Michigan.

File; $8\frac{1}{2}'' \times 34''$; wood: \$4.95

Description: This file, similar to a long wooden box, contains twenty partitions, the spaces between each being of sufficient width to hold eight copies of THE MATHEMATICS TEACHER. Thus, it is adequate for all issues published over a period of twenty years. The file is made of birch plywood except for the slotted pine boards, one in

front and one in back, which hold the partitions in place. It is sent unassembled with a supply of nails.

Appraisal: This inexpensive file will make it possible for the busy teacher to have copies of THE MATHEMATICS TEACHER readily available. It will fit a section of a bookcase or the top of a desk. Sections labeled with a grease pencil will assist filing and locating copy. It will also be suitable for filing other periodicals, pamphlets, or materials of similar size.

FILMS

F. 94—Let's Measure

Coronet Instructional Films, Coronet Building, Chicago 1, Illinois.

Educational collaboration: Dr. F. Lynwood Wren.

First test print: December 1, 1952; Color; 402 feet.

Description: This film touches upon some of the concepts, skills, and uses of linear measurement. Jimmy, an elementary school child, is first seen throwing a baseball. The unseen film commentator states Jimmy's basic problem: He doesn't know how far he has thrown the ball, nor does he know how to find out. The scene shifts to a room in Jimmy's house where Mother is having Jimmy try on one of his last year's spring jackets, which now is much too small. Father enters and guides and directs the measurement experiences which follow. First, they determine (with a foot rule graduated in quarter inches) that Jimmy has grown $\frac{1}{2}$ inch since the last time he was measured. The facts then are established that 12 inches = 1 foot, and that 3 feet or 36 inches = 1 yard. Father introduces the game of "How Long Is That?" Jimmy measures the length of a tabletop and of a windowsill, first estimating the length by laying off "mental images" of a measuring unit (foot or yard). Jimmy next "measures" the height of a table and a chair. He also measures the length of his hand span (6 inches) and of his forefinger (2 inches). The scene shifts to Jimmy's schoolroom where he measures the width of a sheet of paper and of his desk top, using his hand span and forefinger as measuring devices. Flash scenes then show people using or needing linear measurement in three different situations: in lining off a football field, in a lumber yard, and in the dry goods section of a department store. Finally the scene returns to the original setting: Jimmy is throwing a ball on a baseball diamond. The film closes with Jimmy in the *process* of using a yardstick to measure the distance he threw the ball.

Appraisal: No instructional guide was furnished with this test print, thus making valid appraisal more difficult. For instance, the reviewer is uncertain of the exact grade level(s) of the elementary school for which the film is intended. One may infer, however, that it is to be used with children in the very early stages of their study of linear measurement. Regardless, one question arose constantly during each running of the film: What is its purpose? The answer to that question remains in serious doubt. Probably the outstanding feature of the film was its *lack of a clearly identifiable purpose*—or the presence of so *many* intended purposes that none was actually accomplished. Certainly the instructional benefits to be derived from use of the film are not self-evident. Admittedly, numerous important concepts, relationships, and skills regarding linear measurement are included. The fundamental nature of measurement is shown in the repeated application of the measuring unit to the thing to be measured. The use of a "mental image" of the measuring unit in making estimates is shown. The relationships between three measuring units (inch, foot, yard) are established. Attention is directed to the correct method of applying the measuring instrument. The result is inevitable: So many different things are touched in a superficial way at such a rapid pace that little or no clear view of anything is gained. (In this respect the present film is typical of many sister and brother productions in the field of arithmetic.) Finally,

there are several more specific aspects of the film which seem somewhat questionable. The initial measurement experience results in the conclusion that Jimmy has grown $\frac{1}{2}$ inch since he was last measured. Thus, the first measurement is a *fraction* of a unit which at that point is undefined, since the inch has not yet been considered systematically. The foot rule used in making this measurement was graduated in quarter inches, but these graduations were virtually ignored and remained as unexplained markings on the ruler. The yardstick used in making later measurements was graduated in both eighths of an inch and eighths of a yard, again presenting unexplained graduations which easily can lead to confusion. At another point Jimmy "measured" the height of a table and a chair. In each instance he marked the "measurement" on the yardstick with his thumbnail in the customary manner, but—Jimmy never looked at the yardstick to see how high each object was, nor did the commentator give any value for each measurement. Why even bother to "measure"? In the final scene the most important question in the mind of a youngster is left unanswered: Just how far *did* Jimmy throw the ball? The reviewer shares the same disappointment, and views those two words, "The End," with the same frustrated feeling. (Reviewed by J. FRED WEAVER, Boston University.)

INSTRUMENTS

I. 40—Map Projection Demonstrator

Plaxall, Inc., 5–26 46th Avenue, Long Island City, New York.

Model; base; 10″ globe; three projection screens; $75.00.

Description: The base of this device contains a light, switch, transformer and twenty feet of extension cord. The globe which mounts over the light in the base is ten inches in diameter and made of clear plastic. It shows the continents of the world in color. Raised latitude and longitude lines are provided at 10° intervals. The three projection screens of translucent plastic are in the shape of a circular plate, a cylinder, and a cone. By placing these screens tangent to the globe it is possible to demonstrate conic, Mercator, gnomonic, and polar projections.

Appraisal: The principle of each type of map projection, its advantages and disadvantages can readily be demonstrated by this device. The light intensity and color rendition of the projections are clear enough for daytime use. Course lines may be drawn on the surface of the globe with a grease pencil to show how the different types of projections distort a curved line when projected on a flat surface. Although expensive, this well-designed device should make map projections very clear in a short time.

Applications

(Continued from page 439)

Medical experience has shown that the human body does not store up vitamins that are taken in excess of the body's needs. This is contrary to the situation which exists when the body stores excess fat. Thus it would be wasteful to take more than a certain amount of each vitamin. Suppose that, for an infant, five times the MDIR of vitamins is all that can be absorbed.

How much *Tri-Vi-Sol* should be given to supply 5 MDIR of vitamin A? of D?

How much *Gerber's Cereal Food* should be given to supply not more than 5 MDIR for B_1? for B_2?

If the baby likes cereal very well, and eats an ounce of *Gerber's* each day, how much *Tri-Vi-Sol* should be given and still not be wasteful of vitamin B_1? of B_2?

BOOK SECTION

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BOOKS RECEIVED

Elementary

Happy Journey, Beatrice M. Gudridge. Washington, D. C., Department of Elementary School Principals, National School Public Relations Association and National Congress of Parents and Teachers, May 1953. Paper. 32 pp., $0.40.

Learning to Use Arithmetic (Readiness Book), Agnes G. Gunderson and George E. Hollister. Boston, D. C. Heath and Company, 1935. Paper. 64 pp., $0.52.

Learning to Use Arithmetic 1, Agnes G. Gunderson and George E. Hollister. Boston, D. C. Heath and Company, 1953. Paper. 96 pp., $0.64.

Secondary

Business Mathematics Principles and Practice—Complete (Fourth ed.), R. Robert Rosenberg. New York, McGraw-Hill Book Company, Inc., 1953. Cloth. xiv+557 pp., $2.80.

Mathematics in Action, Book 3 (Third ed.), Walter W. Hart and Lora D. Jahn. Boston, D. C. Heath and Company, 1952. Cloth. viii +354 pp., $2.40.

Reviews and Examinations in Algebra (Second ed.), Oswald Tower and Winfield M. Sides. Boston, D. C. Heath and Company, 1953. Cloth. v+183 pp., $2.28.

College

Calculus, C. R. Wylie, Jr. New York, McGraw-Hill Book Company, 1953. Cloth. iii+565 pp., $6.00.

College Algebra, Ross H. Bardell and Abraham Spitzbart. Cambridge, Massachusetts, Addison-Wesley Press, Inc., 1953. Cloth. ix +197 pp., $3.50.

College Algebra, Jack R. Britton and L. Clifton Snively. New York, Rinehart and Company, Inc., 1953. Cloth. x+502 pp., $5.00.

Conformal Mapping, L. Bieberbach, trans. by F. Steinhardt. New York, Chelsea Publishing Company, 1953. Cloth. vi+234 pp., $2.25.

Electric Circuit Theory and the Operational Calculus (Second ed.), John R. Carson. New York, Chelsea Publishing Company, 1953. Cloth. ix+197 pp., $3.95.

Elementary Analysis, Kenneth O. May. New York, John Wiley & Sons, Inc., 1950. Cloth. xi+635 pp., $5.00.

Foundations of the Nonlinear Theory of Elasticity, V. V. Novozhilov. Rochester, New York, Graylock Press, 1933. Paper.vi+233 pp., $4.00.

Nomography and Empirical Equations, Lee H. Johnson. New York, John Wiley & Sons, Inc., 1952. Cloth. ix+150 pp., $3.75.

Principles of Mathematical Analysis, Walter Rudin. New York, McGraw-Hill Book Company, Inc., 1953. Cloth. ix+227 pp., $5.00.

TechniData Hand Book, Edward Lupton Page. New York, The Norman W. Henley Publishing Company, 1942. 64 pp., cloth $1.50; spiral $1.00.

Miscellaneous

Moral and Spiritual Education in Home, School, and Community, National Congress of Parents and Teachers. Chicago, National Congress of Parents and Teachers, 1953. Paper. iii+28 pp., $0.25.

Schoolmarm Abroad with G. I. Jr. (A Three-Year Odyssey), Lula E. Dalton. New York, Exposition Press Inc., 1953. Cloth. 140 pp., $3.00.

Thousands of Science Projects, Margaret E. Patterson and Joseph H. Kraus. Washington, D. C., Science Service, 1953. Paper. 45 pp., $0.25.

BOOKS REVIEWED

Advanced Calculus, Wilfred Kaplan. Cambridge, Massachusetts, Addison-Wesley Press, Inc., 1952. xiii+679 pp., $8.50.

In the opinion of the reviewer this excellent book is an ideal blend of mathematical theory and mathematical practice. It is so filled with applications, examples from the sciences, and illustrative material that it might be thought to be primarily for the student of engineering, science, or applied mathematics; and yet the theory is undiluted and so carefully and concisely written as to meet the needs of any student. Definitions and theorems are correctly stated and either proved or adequate references to the literature given. In every case numerous examples and applications stimulate interest in and illustrate the theory. The book has many secondary but important virtues as well; specifically, a most complete bibliography together

with numerous textual references to the literature, a supply of good problems, and unusually good diagrams. In outline the book comprises five chapters on differential and integral calculus of functions of several variables (with emphasis on vector methods), a chapter on series, sequences, etc., a chapter on Fourier series, two very good chapters on ordinary and partial differential equations, and finally an extensive introduction to analytic functions of a complex variable. It need hardly be added that the reviewer considers this a valuable addition to the textbooks on advanced calculus.—WILLIAM M. BOOTHBY, Northwestern University, Evanston, Illinois.

Contributions to the Founding of the Theory of Transfinite Numbers, Georg Cantor. New York, Dover Publications, Inc., 1952. ix+211 pp., \$1.25 (paper) or \$2.75 (cloth).

This book, previously published by another company at least as early as 1915, is a translation of two articles from the *Mathematische Annalen* (1895 and 1897). It also contains an introduction of eighty-two pages by Jourdine.

In this edition, no mention is made of the earlier publication date. However, this edition appears to be identical with the 1915 version, including the same minor errors.

Outside of historical value, most of the sheen has been removed from this volume since its first publication.—T. C. LITTLEJOHN, Northwestern University, Evanston, Illinois.

The Design and Analysis of Experiments, Oscar Kempthorne. New York, John Wiley & Sons, Inc., 1952. xix+631 pp., \$8.50.

Professor Kempthorne approaches the subject of the text from a very theoretical viewpoint and, as a result, the book has a high degree of mathematical rigidity. It is not for the beginning student in mathematical statistics, but is excellent for anyone with a good mathematics background who is interested in why the tools of experiment design and analysis work. Of course, a thorough understanding of the "why" will lead to more versatility in their use.

The book covers such a broad range that it is best digested a little at a time. It is probably more useful as a reference than as a text.—WILLIAM E. FELLING, Parks College of Aeronautical Technology, East St. Louis, Illinois.

Elementary Differential Equations, Earl D. Rainville. New York, The Macmillan Co., 1952. xii+392 pp., \$5.00.

Of all standard topics covered in undergraduate mathematics, that one having the best textbooks available is probably differential equations. Rainville's new book on the subject compares very favorably with its select company. Besides its clear, readable style, noteworthy features include the large number of graded problems (with reliable answers) occurring after nearly every section, the author's method of "solving" simple recurrence relations, and the early introduction of the problem of finding a differential equation satisfied by each member of a family of curves, a move which tends to give meaning to the arbitrary constants found in solutions of differential equations.

With a few additions, this book would make a good text for engineering as well as liberal arts courses in differential equations. For example, all the machinery is included that is necessary in order to solve such equations as the hypergeometric, Bessel's, and Legendre's equations, but discussion of those equations is omitted. Also, more practical applications and applied problems are needed, even for a liberal arts course (the author's reply to the last remark is contained in the preface of his book).

The first thirteen chapters are reproduced from a previous text by Rainville, *A Short Course in Differential Equations*. The material added includes power series solutions, numerical methods, partial differential equations, and the application of Fourier series to boundary value problems.—T. C. HOLYOKE, Northwestern University, Evanston, Illinois.

Foundations of Combinatorial Topology, L. S. Pontryagin. Rochester, New York, Graylock Press, 1952. Paper, xii+99 pp., \$3.00.

In this short book, a translation from the 1947 Russian edition, the author defines the homology groups of a complex and proves the fundamental invariance theorem (i.e., that the homology groups of homeomorphic complexes are isomorphic). In addition he discusses the concept of homotopy classes of mappings, dimension of a topological space, and fixed point theorems, the latter quite thoroughly. In this short space the author has presented in his usual elegant style some of the main content of the homology theory of complexes. The book requires some maturity on the part of the reader together with some knowledge of group theory and point set topology. No illustrative material is included as the book was apparently intended to complement other texts on the subject such as that of Alexandrov and Efrimovitch.—WILLIAM M. BOOTHBY, Northwestern University, Evanston, Illinois.

Fundamental Procedures of Financial Mathematics, Merrill Rassweiler and Irene Rassweiler. New York, The Macmillan Co., 1952. vii+260 pp., \$3.25.

The material included in this text was tried out for several semesters in the author's classes in the general college of the University of Minnesota. The text was designed to treat the usual material in business mathematics in a manner which would require little background knowledge in formal mathematics. The author believes that much of this material can be understood with a background based upon arithmetic alone. Throughout the text, therefore, algebraic language is avoided and the usual algebraic formulas are not used.

The procedures used in the solution of problem material are carefully outlined in "step" form and the computation of the examples are carefully worked out in a parallel manner step by step. Data used for the problems are based on pricing obtained from tables of operation published by national business organizations.

Topical material in the first portion of the text includes the usual business problems based upon percentage such as commission, pricing and profits, taxation, discounts, simple interest and discount, and various types of term insurance. The second portion of the text includes topics based upon compound interest such as the various types of annuities, with separate chapters on amortization, bonds, life annuities and life insurance.

While the text does not contain the usual tables necessary for the solution of annuities, this would not seem to be a serious omission since many instructors prefer to have their students purchase these tables separately.

The text seems to be well written with adequate material for a two semester three hour course. If the formula type of problem solution seems to be desirable for a given class it would seem that their introduction by the teacher would present little difficulty. The text merits consideration especially for a class which has a minimum of mathematical background.—HERBERT HANNON, Western Michigan College of Education, Kalamazoo, Michigan.

Introduction to the Foundations of Mathematics, R. L. Wilder. New York, John Wiley & Sons, Inc., 1952. xiv+305 pp., $5.75.

In this excellent work the author manages to catch much of the spirit of the course which he has given for over twenty years. The book is very readable; the illustrative material is so chosen that a reader with very little mathematical background can understand it, and yet one is carried to the threshold of some of the difficult problems of the foundations of mathematics. The axiomatic method is thoroughly discussed, as is the theory of sets, including cardinal and ordinal numbers, well-ordering, and Zorn's lemma. There are chapters on the real numbers and the continuum and on elementary group theory. Finally, nearly half the book is devoted to various viewpoints on the foundations of mathematics, i.e., intuitionism, formalism, etc., including a chapter expounding the personal views of the author.—WILLIAM M. BOOTHBY, Northwestern University, Evanston, Illinois.

Linear Integral Equations, W. V. Lovitt. New York, Dover Publications Inc., 1950. ix+250 pp., $3.50.

This is the same edition that was first published in 1924. It is virtually a reprint of notes of lectures by Bolza given at the University of Chicago in 1913 (this the author acknowledges in the preface). Even at the time of first publication, the discussion was rather inadequate. The bibliography is very short, only seven entries.

The contents are arranged in six chapters. In the first three chapters the integral equation is considered as a generalization of an algebraic problem and the Fredholm theory is developed. Chapter four deals with applications, including the Dirichlet and Neumann problems. In chapter five, Lovitt discusses the symmetric kernels and orthogonal functions. Chapter six is devoted to applications in ordinary differential equations, the calculus of variations, and heat flow.

It appears as though the author wrote the book purposely avoiding the use of the theory of functions of a complex variable, preferring to establish longer proofs algebraically. Too, there are several misprints in the book, but they should cause the reader little difficulty.

Although the treatment is not exhaustive, the book is very readable and treats the subject in an interesting fashion. In spite of its limitations, the book would be a good text. To the reviewer's knowledge (and the publisher's claim), this book is still the only "text on linear integral equations in the English language."—T. C. LITTLEJOHN, Northwestern University, Evanston, Illinois.

Mathematics of Finance (Third ed.), Thomas M. Simpson, Zareh M. Pirenian, and Bolling H. Crenshaw. New York, Prentice-Hall, Inc., 1951. xv+335 pp., $4.75.

This third edition of a very successful textbook in the field of mathematics of finance retains all of the excellent features of the previous editions. Revision consists mainly in the replacement of the problems by new ones graded as to difficulty, and greater in number and variety.

A minimum of one year of high school algebra is required if the textbook is to be used in its entirety, while in classes where more algebra is required as a prerequisite the first chapters may be omitted.

In addition to the topics usually included in a textbook on mathematics of finance a chapter on elementary statistics is included. Interest rates and exercises are revised to conform more nearly to recent trends and symbolism used conforms to the notation approved by the National Congress of Actuaries. The problems in life insurance are still based upon the American Experience table however.

In actual classroom use the reviewer has found this textbook to be easily understood by the students and very usable both from the standpoint of completeness of material and manner of presentation.—HERBERT HANNON, Western Michigan College of Education, Kalamazoo, Michigan.

Practical Calculus (Rev. Second ed.), Claude I. Palmer and Claude E. Stout. New York, McGraw-Hill Book Co., 1952. xx+470 pp., $6.00.

This book is designed to give the person with little mathematical training a thorough understanding of how calculus can be applied to practical problems. By means of numerous examples

and careful explanations the authors have made a serious attempt to write a calculus text to be used by a student without benefit of classmates or teacher. Practically all of the topics usually studied in a first year calculus course are included. The treatments of triple integration and infinite series are somewhat sketchy. Throughout the text the interpretation of the definite integral as the increment of the antiderivative receives considerable emphasis. (The interpretation as a limit of a Riemann sum appears late in the text.) In this connection the differential receives more than the usual amount of attention. Although the ideas are clarified by examples, the formalized rules and definitions are not always clearly worded. After discussing the idea of one variable changing uniformly or nonuniformly with respect to another the authors state that there are three ideas related to a train gaining speed after leaving a station: "(1) the change or increment of time, (2) the actual change or increment in the distance, (3) the change in the distance if that change had become and remained uniform." This is confusing if the reader identifies "that change" in (3) as referring to any one of the changes previously mentioned in the three ideas. Instantaneous rate of change is not taken up formally until the next chapter. The wording of the definition, "The differential of the dependent variable is what would be its increment, if at the corresponding values considered, its change became and remained uniform with respect to the independent variable" also could be improved. Undoubtedly the authors have made a worthwhile contribution in producing a text for a special purpose. Many instructors would reject it as a suitable college text because of the arrangement of topics and, perhaps, because some proofs are omitted.—LAWRENCE A. RINGENBERG, Eastern Illinois State College, Charleston, Illinois.

Science and Hypothesis, Henri Poincaré. New York, Dover Publications, Inc., 1952. xxvii +244 pp., $1.25 (paper) or $2.50 (cloth).

This is a republication of the 1905 English translation. It is essentially a philosophical examination of the foundations of mathematics and physics.

The first part of the book expounds the modern axiomatic approach to mathematics and clarifies the relation of geometry to physics in a manner still acceptable. As usual, Kant turns out to be irrelevant. It is curious that the concept of the Dedekind cut is here attributed to Kronecker.

Since the concept of theoretical physics as description rather than explanation did not flower in earnest until the advent of relativity and quantum theory, it is of note that it was so clearly premeditated here by Poincaré. The book is important insofar as it represents an intermediate stage in the development of the present-day scientific epistemology. In a few passages a modern physicist will turn slightly pale, but any reader will "survive" with profit.—W. E. JENNER, Northwestern University, Evanston, Illinois.

Science and Method, Henri Poincaré. New York, Dover Publications, Inc., 1952. 288 pp., $1.25 (paper) or $2.50 (cloth).

This book can be described as a collection of essays on mathematics and physics, predominantly the former, and contains some of the author's finest writings. One of the main themes of the book is the inquiry into what are the important things to do in mathematics. These comments of Poincaré are most relevant in these axiomatic days. Of particular interest are the author's views on the future of mathematics (with which it is interesting to compare the present state of the science) and the psychology of invention in mathematics. The comments on the work of Hilbert and Russell on the foundations are infused with quite vitriolic sarcasm in places. Although these are amusing to the reader, they, nevertheless, possess a quite serious content to which it would be well to pay heed.

One chapter on the teaching of elementary mathematics contains much profound pedagogical wisdom. The book should be read by any student, teacher, or research worker in mathematics. Its republication is most timely.—W. E. JENNER, Northwestern University, Evanston, Illinois.

Space—Time—Matter, Hermann Weyl (translated from the German by Henry L. Brose). New York, Dover Publications, Inc., 1950. xviii +330 pp. $3.95.

This is the first printing in this country of the 1922 English translation of *Raum, Zeit, Materie* by one of the greatest living mathematicians. The book is addressed to specialists in theoretical physics and assumes a very extensive background in mathematics and physics on the part of the reader.

One undergoes the customary experience in reading one of Professor Weyl's books—the style of writing is so agreeable that it comes as something of a shock when the reader suddenly finds that he does not understand what is being said.

Of particular interest to mathematicians is the extensive account of Riemannian geometry. The author's profound insight into this subject renders the book invaluable. Differential geometers should have no difficulty in rendering more "respectable," in the modern sense, the infinitesimals that are used. But this is not submitted as a criticism since the methods we have at present did not exist when the book was written.

There is included a new preface by the author relating the book to developments in physics up to 1950.—W. E. JENNER, Northwestern University, Evanston, Illinois.

Using Mathematics, David H. Patton and William E. Young. Syracuse, Iroquois Publishing Co., Inc., 1952. xx +587 pp. $2.52.

Using Mathematics is intended for a general course in the ninth year, combining arithmetic, algebra, geometry, and trigonometry and showing their use in everyday life. The chapters on

occupational arithmetic, insurance, taxes, budgets, business, etc., have a quantity of application material that would appeal to many students. There seems, however, to be more higher finance than a ninth-grade student will ordinarily appreciate. Much of that material would be better in the twelfth grade.

In the chapters on algebra there seems to be entirely too much stating of rules and "how to do's" without much encouragement of real thinking on the part of the student. Many of the rules tell the student to "write after," "annex to," "write over," "cancel," "prefix," which expressions certainly do not train the student to think of the mathematical operations he is performing. There is a tendency to make a separate rule for each little case, adding to the bewildering array of rules.

The geometry chapters are much like the beginning of a conventional geometry book except that they are less rigorous and more confusing.

There is a section reviewing fundamentals of arithmetic in the back of the book which consists of rules and exercises.

The binding of the book makes it very difficult to hold it open to the desired page. The margins are narrow and the print is such that rules, descriptive material, and exercises all seem to run together and it is hard to tell where a new idea starts. The Table of Contents is very extensive but hard to read. The book is well illustrated.—Henry Swain, New Trier Township High School, Winnetka, Illinois.

Working With Numbers, Book 1 (Teacher's text edition), Joyce Benbrook, Cecile Foerster, and James T. Shea. Austin, Texas, The Steck Company, 1952. iv+80 pp.+46 pp. manual, $1.72.

Working With Numbers, Book 1 (Teacher's worktext edition), same authors as above. Austin, Texas, The Steck Company, 1952. iv+112 pp.+37 pp. manual, $0.64.

The outstanding features of both books are: (1) the sound philosophy based on the meaning theory; (2) the carefully developed readiness program; (3) the complete coverage of the beginning number program; (4) the great amount of work done before any printed vocabulary is introduced; (5) the unusually high quality of color illustrations, paper and printing.

Either book could be used alone (the worktext is complete within itself) or they could be used as companion books. The vocabulary has been especially well controlled. Children meet only forty-three words in the entire program. The methods of giving children experiences with measurement, developing mathematical vocabulary and putting meaning into numbers are varied, thorough, and methodical. Teachers surely would not lack for material; in fact, they might find the book adaptable for use partly in grade one and continued in grade two.—Irene M. Larson, Director of Elementary Instruction, Green Bay, Wisconsin.

Working With Numbers, Book 2 (Teacher's text edition), Joyce Benbrook, Cecile Foerster, and James T. Shea. Austin, Texas, The Steck Company, 1952. Cloth. 111 pp.+46 pp. manual, $1.88.

Working With Numbers, Book 2 (Teacher's worktext edition), same authors as above. Austin, Texas, The Steck Company, 1952. Paper. 128 pp.+38 pp. manual, $0.64.

These books are attractive, well planned, and colorful, nicely adapted to second grade level; teachers should find them easy and profitable to use. The "plus" pages mentioned above are pages of directions and suggestions to teachers for use with these books. The other pages are the actual text which is to be placed in the child's hands.

The development throughout is both logical and psychological; that is, both the development of the child and the development of the number work have been thoughtfully considered. For example, the books begin with an appealing review chapter entitled "Do You Remember?" in which the work of Book 1 is recalled; this "Do You Remember" idea is continued throughout the books as a bit of maintenance work, and at the end as a summary. The authors have recognized that concrete experiences with number must precede the learning of general or abstract number, and in the manual have advised teachers to provide such experiences.

Careful attention has been given to the development of the "tens" idea and its meaning in the number system; addition and subtraction have been presented by "adding 1," and "taking away 1"; later the other two subtraction ideas are also introduced in a meaningful way; multiplication is touched on through the "doubles" in addition. Fractions ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$) are presented as meaning a part of a whole and also as a part of a group. The illustrations used to help with fraction meanings seem to be unusually numerous and meaningful.

Another unusual feature is the work on making change; these books do not feature this operation as subtraction, but use it as a "counting on" operation—an idea in line with the actual practice in making purchases and receiving change.

Work in simple measures, telling time, and applications of the fundamental operations are brought in at various appropriate places, each with illustrations that make the work understandable to children.

One criticism: How can teachers find time to do justice to both these books in a single year? Either one would make a good course: the text offers more ideas for presenting material, and the worktext offers more child activity.—Elinor B. Flagg, Illinois State Normal University, Normal, Illinois.

Program
The Fourteenth Christmas Meeting
The National Council of Teachers of Mathematics

University of California, Los Angeles, California
December 28, 29, 30, 1953

Convention Theme: The Mathematics Teacher in the Twentieth Century

MONDAY, DECEMBER 28

8:00 A.M.–5:00 P.M. **Registration**—Foyer of Business Administration-Economics Building

9:00–10:30 A.M. **General Session**—Room 147

Welcome, RAYMOND ALLEN, Chancellor, University of California, Los Angeles, California

Response, MARIE S. WILCOX, Vice-president, The National Council of Teachers of Mathematics, Thomas Carr Howe High School, Indianapolis, Indiana

Common Goals for the National Council and for Mathematics Teachers, JOHN R. MAYOR, President, The National Council of Teachers of Mathematics, The University of Wisconsin, Madison, Wisconsin

10:45 A.M.–12:00 NOON. **Sectional Meetings of General Interest**

Section I—Room 147

Recent Research and Its Implications for the Classroom, DALE CARPENTAR, Los Angeles City Schools, Los Angeles, California

Current Trends in the Teaching of Secondary School Mathematics, LUCIEN B. KINNEY, Stanford University, Stanford, California

Section II—Room 121

Meteorites, Captives from Space, FREDERICK C. LEONARD, University of California, Los Angeles, California

Some Applications of Mathematics in the Aircraft-Electronics Industry and Their Relation to Instruction, J. H. RUBEL, Hughes Aircraft Company, Culver City, California.

Section III—Room 191

Panel Discussion: *What Mathematics Are We Teaching at the Various Grade Levels?*

Primary: RUTH ROWLAND HALL, Pasadena Public Schools, Pasadena, California

Upper Grades: LOLITA PETERSON, Oakland Public Schools, Oakland, California

Junior High School: AGNES HERBERT, Clifton Park Junior High School, Baltimore, Maryland

Senior High School: MARTHA HILDEBRANDT, Proviso Township High School, Maywood, Illinois

Junior College: ARTHUR J. HALL, San Francisco State College, San Francisco, California

Moderator: JESSE BOND, University of California, Los Angeles, California

1:30–2:45 P.M. **Sectional Meetings**

Elementary Section—Room 50

The Role of Insight in the Learning of Mathematics at the Elementary School Level, HENRY VAN ENGEN, State Teachers College, Cedar Falls, Iowa

Secondary School Section—Room 161

The Role of Insight in the Learning of Mathematics at the Secondary School Level, HOWARD F. FEHR, Teachers College, Columbia University, New York City

Junior College Section—Room 191

The Relationship between Mathematics, Engineering, and Other Technical

Fields in the Junior College, ERNEST B. EILERTSEN, City College of San Francisco, San Francisco, California

Terminal Mathematics in the Junior College, FRED MARER, Los Angeles City College, Los Angeles, California

3:00–4:00 P.M. **Discussion Groups**

Group 1. Room 178

Topic: *The Teaching Major and Minor in Mathematics*

Leader: CLIFFORD BELL, University of California, Los Angeles, California

Group 2. Room 167

Topic: *Field Work in Senior High School Mathematics*

Leader: F. F. SCHEPMAN, North Bend High School, North Bend, Oregon

Group 3. Room 162

Topic: *Going from the Concrete to the Abstract with Trigonometric Functions*

Leader: EDWIN EAGLE, San Diego State College, San Diego, California

Group 4. Room 154

Topic: *The Problems of Teaching Mathematics in a Small High School*

Leader: TOM W. PREECE, Trinity County High School, Weaverville, California

Group 5. Room 147

Topic: *In Quest of Unifying Principles: I. Proportion*

Leader: SAMUEL E. URNER, Los Angeles State College, Los Angeles, California

Group 6. Room 170

Topic: *The Examination and Evaluation of Objectives of Algebra Instruction in Secondary Education*

Leader: KEITH SMITH, Los Angeles City Board of Education, Los Angeles, California

Group 7. Room 161

Topic: *An Approach in Treating Problems in Consumer Mathematics*

Leader: LOIS M. MAYOR, Monrovia-Duarte High School, Monrovia, California

Group 8. Room 146

Topic: *Problems in Supervising a Mathematics Program*

Leader: ELIZABETH ROUDEBUSH, Seattle Public Schools, Seattle, Washington

Group 9. Room 121

Topic: *Group Dynamics, a Potent Factor in the Teaching and Learning of Informal Geometry in the Junior High School*

Leader: MARY C. ROGERS, Roosevelt Junior High School, Westfield, New Jersey

Group 10. Room 50

Topic: *Problem Solving in the Early Elementary Grades*

Leader: AGNES GUNDERSON, The University of Wyoming, Laramie, Wyoming

3:00–4:00 P.M. **Elementary Laboratory**—Room191

Director: ELDA L. MERTON, Chicago, Illinois

4:15 P.M. Sightseeing Trips

TUESDAY, DECEMBER 29

9:30–10:30 A.M. **General Session**—Room 147

Inservice Growth in Meaningful Teaching, RICHARD MADDEN, San Diego State College, San Diego, California

10:45 A.M.–12:00 NOON. **Sectional Meetings**

Elementary Section—Room 50

Panel Discussion: *Making Arithmetic Function in the Lives of the Learner*

Chairman: SARAH A. RHUE, The University of Wisconsin, Madison, Wisconsin

Participants: AGNES GUNDERSON, The University of Wyoming, Laramie, Wyoming; J. MAURICE KINGSTON, University of Washington, Seattle, Washington; EDWENA MOORE, San Diego County Schools, San Diego, California; LOIS TRAINOR, Pasadena Public Schools, Pasadena, California

Junior High School Section—Room 146

Panel Discussion: *Mathematics for the Average Citizen*

Chairman: JOHN R. EALES, Los Angeles County Schools, Los Angeles, California

Participants: Other panel members will

be from business and industry in California

Geometry Section—Room 161

Modern Postulates of Geometry, PAUL H. DAUS, University of California, Los Angeles, California

Let's Teach Geometry to All Tenth Grade Students, ELIZABETH ROUDEBUSH, Seattle Public Schools, Seattle, Washington

Teacher Education Group—Room 121

Converting Number Scores into Grades, C. B. READ, The Municipal University of Wichita, Wichita, Kansas

The Place of Mathematical Recreations in Elementary Arithmetic Instruction, RAYMOND C. PERRY, The University of Southern California, Los Angeles, California

College Section—Room 191

The Non-Directive Teaching of College Mathematics, ROBERT S. FOUCH, Arizona State College, Tempe, Arizona

General Mathematics Goes to College, ANNA S. HENRIQUES, University of Utah, Salt Lake City, Utah

Section Sponsored by the Colorado Council of Teachers of Mathematics—Room 162

Panel Discussion: *What Can be Done for the Talented in Mathematics?*

Chairman: H. W. CHARLESWORTH, East High School, Denver, Colorado

Participants: RUTH H. TUTTLE, Palmer Elementary School, Denver, Colorado; BURNETT SEVERSON, Morey Junior High School, Denver, Colorado; AMANDA A. LINDSEY, West High School, Denver, Colorado; OTHO M. RASMUSSEN, University of Denver, Denver, Colorado

1:30–2:45 P.M. **Sectional Meetings**

Elementary School Section—Room 50

Panel Discussion: *The In-Service Training of Teachers of Elementary Mathematics*

Chairman: MAUDE COBURN, Oakland Public Schools, Oakland, California

Participants: LESTE HOEL, Portland Public Schools, Portland, Oregon; EDNA WISELY, Ventura County Schools, Ventura County, California; MAX MILLER, San Diego City Schools, San Diego, California; MARGUERITE BRYDEGAARD, San Diego State College, San Diego, California

Enrichment Section—Room 161

Mathematical Recreations, LOUIS GRANT BRANDES, Encinal High School, Alameda, California

Enrichment of Mathematics Teaching, SOPHIA L. McDONALD, University of California, Berkeley, California

Evaluation Section—Room 178

Evaluation of Pupil Growth, MARIAN CLIFFE, Verduga Hills High School, Glendale, California

Evaluation of a School District Mathematics Program, ANNA L. DAVIS, Pasadena City Schools, Pasadena, California

Curriculum Section—Room 162

Modern Trends in Mathematics Curriculum, JACK D. WILSON, San Francisco State College, San Francisco, California

Some Considerations in Curriculum Planning, Sister M. MADELINE ROSE, College of Holy Names, Oakland, California

Applications Section—Room 121

Some Applications of Mathematics, L. J. ADAMS, Santa Monica City College, Santa Monica, California

Mathematicians and Automata, PAUL BROCK, Consolidated Engineering Corporation, Pasadena, California

College Section—Room 147

Some Elementary Problems in the Calculus of Variations, MAGNUS R. HESTENES, University of California, Los Angeles, California

A Problem in Theory of Equations and Calculus, KENNETH W. WEGNER, Carleton College, Northfield, Minnesota

Section Sponsored by the Arizona Mathematics Association—Room 154

Numbers for Slow Learners, RUTH COOK, Phoenix, Arizona

More Meaningful Experiences as a Substitute for Drill, HELEN DREYFUS, Whittier School, Phoenix, Arizona

Vitalizing General Mathematics, J. S. FLIPPER, Carver High, Phoenix, Arizona

3:00–4:30 P.M. **Mathematics Laboratories**

Elementary School Laboratory—Room 191

Director: ELDA L. MERTON, Chicago, Illinois

Junior High School Laboratory—Room 167

Director: LAUREN G. WOODBY, State Teachers College, Mankato, Minnesota

Senior High School Laboratory—Room 170

Director: RACHEL P. KENISTON, Stockton College, Stockton, California

3:00–5:00 P.M. **Films**—Room 146

Chairman: WILLIAM H. GLENN, Jr., Pasadena Public Schools, Pasadena, California

6:00 P.M. **Banquet**—Kerckhoff Hall

Speaker: L. M. K. BOELTER, Dean of the College of Engineering, University of California, Los Angeles, California

WEDNESDAY, DECEMBER 30

9:00–10:00 A.M. **General Session**—Room 147

The Mathematics Teacher and Public Relations, C. C. TRILLINGHAM, Superintendent of Schools, County of Los Angeles, Los Angeles, California

10:15 A.M.–12:00 NOON **Sectional Meetings**

Elementary School Section—Room 50

Language and Meaning in Arithmetic, R. L. MORTON, Ohio University, Athens, Ohio

Zero Difficulties in Teaching Multiplication, LOIS F. HARVEY, San Diego City College, San Diego, California

Meaningful Instruction on Fraction Concepts and Computations, CHARLES F. HOWARD, Sacramento State College, Sacramento, California

Junior High School Section—Room 146

Developing a Mathematics Program Which Will Serve the Needs of All Types of High School Students, GEORGE V. HALL, San Diego City Schools, San Diego, California

Improvement of Reading in the Mathematics Class, L. CLARK LAY, John Muir College, Pasadena, California

Visual Aids for Functional Mathematics, ALICE PHILLIPSON, Los Angeles Public Schools, Los Angeles, California

Senior High School Section—Room 161

Developing Meanings in Algebra, JEAN TULLEY, Stockton College, Stockton, California

Technical Mathematics in Grades 9, 10 and 11, MYRON F. ROSSKOPF, Teachers College, Columbia University, New York City

Developing Meanings in High School Mathematics through Visual Aids, RACHEL P. KENISTON, Stockton College, Stockton, California

Junior College Section—Room 191

Business Arithmetic at the Junior College Level, FRED L. BEDFORD, Phoenix College, Phoenix, Arizona

Confusion of Language and Notation in Elementary Mathematics, W. W. MITCHELL, Jr., Phoenix College, Phoenix, Arizona

Visual Techniques in Trigonometry, CLELA D. HAMMOND, El Camino College, El Camino College, California

Teacher Education Section—Room 121

Some Problems in Student Teaching Programs in Mathematics, GILBERT ULMER, University of Kansas, Lawrence, Kansas

Let's Train Teachers to Teach Non-College Preparatory Mathematics, R. P. ABBOTT, Frick Junior High School, Alameda, California

History of Reports on Mathematics in Retrospect and Prospect, C. RICHARD PURDY, San Jose State College, San Jose, California

Section Sponsored by the New Mexico Mathematics Section of the New

Mexico Education Association—Room 154

Panel Discussion: *Coordination of High School and College Mathematics in New Mexico*

Chairman: OLIVE WHITEHILL, Deming, New Mexico

Participants: FRANK C. GENTRY, University of New Mexico, Albuquerque, New Mexico; NELSON HAGGERSON, New Mexico Military Institute, Rosewell, New Mexico; MAX KRAMER, New Mexico College of Agriculture and Mechanic Arts, State College, New Mexico; JOHN HARTY, New Mexico Institute of Mining and Technology, Socorro, New Mexico; JERALD LONGBOTHAM, Socorro High School, Socorro, New Mexico; JAMES L. PHARRIS, Gallup Junior High School, Gallup, New Mexico.

12:30 P.M. **Luncheon**—Kerckhoff Hall

The Faculty and the Mathematics Teacher, HAROLD P. FAWCETT, Ohio State University, Columbus, Ohio

Program Chairman: Marie S. Wilcox, Thomas Carr Howe High School, Indianapolis, Indiana.

The program chairman wishes to make grateful acknowledgment of the assistance given by the following people in making suggestions as to speakers and topics for this program: Arthur J. Hall and the Policy Committee of the California Mathematics Council, Clifford Bell, Harold M. Bacon, Norman Arnt, Ida May Bernhard, Marion T. Bird, W. A. Brownell, Marguerite Brydegaard, Maude Coburn, Edwin Eagle, Maurice Hartung, George C. Kyte, Eleanor Lazansky, John R. Mayor, Irene Sauble, and Peter L. Spencer.

ANNOUNCEMENTS

Registrations: The registration fee is fifty cents for members of the National Council of Teachers of Mathematics, members of the Mathematical Association of America, and for teachers in elementary schools. The fee for nonmembers and visitors is $1.50. Undergraduate students sponsored by a faculty member, relatives of members, invited speakers who are not members, members of the press, and commercial exhibitors are not charged the registration fee but should register. You are urged to register in advance, but, if you do, please check in at the Registration Desk at your arrival at the meeting. Use the Advance Registration and Reservation Form supplied herewith. Registration headquarters will be in the foyer of the Business Administration-Economics Building.

Hotel Reservations: A limited number of persons can be accommodated at Hershey Hall, a dormitory on campus. This is convenient, being situated very close to the meetings. There are twin beds in each room. The rate is $2.50 per day per person. A very few single rooms are available at $4.00. No meals are served, but the university cafeteria will be open, and there are several attractive restaurants in the vicinity. Reservations will be assigned in order of receipt, so the first 75 to apply will receive these accommodations. The dates that Hershey Hall will be available are from December 27 to December 30, with departure by late afternoon of December 30.

There are many motels and apartment hotels of outstanding merit in the not-so-immediate vicinity of the university. Reservations at these places must be made directly with the motel or hotel. A list will be furnished upon request. Since this is Rose Bowl and Rose Parade time, reservations should be made and confirmed early.

Banquet and Luncheon Reservations: Reservations for the banquet on Tuesday and the luncheon on Wednesday should be made in advance. Requests should be accompanied with check or money order. All orders received before December 15 will be acknowledged. Banquet, $3.00; luncheon, $1.75. Use the Advance Registration and Reservation Form shown on page 458.

Entertainment: Sightseeing trips, visits

ADVANCE REGISTRATION AND RESERVATION FORM

Fill out completely and mail to Mr. John F. Williams, John Muir College, Pasadena, California, before December 15, 1953.

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Members of families and friends of those attending the meetings are welcome to all social and recreational events.

Registration Fee: $0.50 or $1.50 Amount $________

________ Banquet Reservations at $3.00 each ________

________ Luncheon Reservations at $1.75 each ________

________ Room Reservations, Hershey Hall ________

Total $________

If list of nearby motels and hotels is desired, check here ☐
Make checks payable to: Dr. Marian Cliffe, Treasurer.

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to radio and television centers, and other entertainment are being arranged for visitors to the meeting.

Supplies and Equipment: Speakers and other participants on the program who need blackboards, projection equipment, or other materials should communicate with the local chairman, Dr. Clifford Bell, the University of California, Los Angeles, before December 10.

Exhibits: Commercial exhibits of textbooks and teaching aids will be in the foyer of the Business Administration-Economics Building. Teachers are invited to bring exhibits of classroom work. Inquiries concerning exhibits should be addressed to L. Clark Lay, John Muir College, Pasadena, California.

Films and Filmstrips: Mathematics films and filmstrips will be shown as indicated in the program in Room 146 of the Business Administration-Economics Building. Requests or suggestions regarding films should be addressed to William H. Glenn, Pasadena City Schools, 351 South Hudson Avenue, Pasadena 5, California.

Refunds on Reservations: No ticket refunds will be made later than three hours prior to the function for which the reservation was made.

You Will Want to Send for This

The United States Department of Health, Education, and Welfare has just released Bulletin 1952, No. 15 entitled "Education for the Talented in Mathematics and Science." This pamphlet is a report of a Joint Conference of the Cooperative Committee on the Teaching of Science and Mathematics of the American Association for the Advancement of Science and the United States Office of Education.

The report is helpful. It considers such topics as: (1) Who are the talented in mathematics and science? (2) Ways of identifying the talented, and (3) Providing for the talented in science and mathematics. It can be obtained from the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. Catalog No. FS 5.3:952/15. Price 15 cents.

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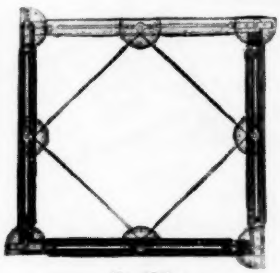

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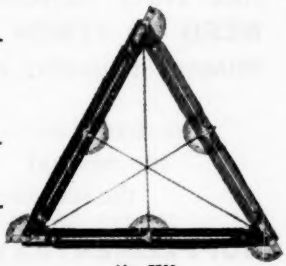

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- SELECTED TOPICS IN TEACHING MATHEMATICS, *Third Yearbook*, 1928. 284 p. $1.00.

  SIGNIFICANT CHANGES AND TRENDS IN THE TEACHING OF MATHEMATICS THROUGHOUT THE WORLD SINCE 1910, *Fourth Yearbook*, 1929. 196 p. $1.00.

  MATHEMATICS IN MODERN LIFE, *Sixth Yearbook*, 1931. 204 p. $1.00.

  THE TRAINING OF MATHEMATICS TEACHERS IN SECONDARY SCHOOLS, *Fourteenth Yearbook*, 1939. 246 p. $1.00.

  THE PLACE OF MATHEMATICS IN SECONDARY EDUCATION, *Fifteenth Yearbook*, 1940. 269 p. $3.00.

  ARITHMETIC IN GENERAL EDUCATION, *Sixteenth Yearbook*, 1941. 347 p. $3.00.

  MULTI-SENSORY AIDS IN TEACHING MATHEMATICS, *Eighteenth Yearbook*, 1945. 460 p. $3.00.

  SURVEYING INSTRUMENTS, HISTORY AND CLASSROOM USE, *Nineteenth Yearbook*, 1947. 424 p. $2.00.

  THE METRIC SYSTEM OF WEIGHTS AND MEASURES, *Twentieth Yearbook*, 1948. 317 p. $2.00.

  THE LEARNING OF MATHEMATICS, ITS THEORY AND PRACTICE, *Twenty-first Yearbook*, 1953. 364 p. $4.00. $3.00 to members of the Council.

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